HSB
Hochschule Bremen
City University of Applied Sciences

INSTITUTE OF
WATERACOUSTICS,
SONAR ENGINEERING AND
SIGNAL THEORY

Underwater Acoustics and Sonar Signal Processing

Contents

1 Fundamentals of Ocean Acoustics
2 Sound Propagation Modeling
3 Sonar Antenna Design
4 Sonar Signal Processing
5 Array Processing

1 Fundamentals of Ocean Acoustics

The ocean is an extremely complicated acoustic medium. The most characteristic feature of the oceanic medium is its inhomogeneous nature. There exist two kinds of inhomogeneities

- regular and
- random.

Both strongly influence the sound field in the ocean, where

- regular variations of the sound velocity versus depth lead to the formation of an underwater sound channel
- random inhomogeneities give rise to scattering of sound waves and, therefore, to fluctuations in the sound field.

In the following sections we will consider in more detail the

- Sound Velocity in the Ocean
- typical Sound Velocity Profiles
- Attenuation of Sound
- Ocean surface and bottom interaction
- Sound Propagation
- Sonar Equation (Performance Prediction)

1.1 Sound Velocity in the Ocean

Variations of the sound velocity c in the ocean are relatively small. As a rule, c lies between about 1450 m/s and 1540 m/s.

Even though the changes of c are small, i.e. $\leq \pm 3\%$, the propagation of sound can be significantly effected.

The sound velocity can be directly measured by velocimeters or calculated by empirical formulae if the

- temperature (T)
- salinity (S)
- hydrostatic pressure (P) or depth (z)

are known.

The measurement error of modern velocimeters is < 0.1 m/s.

The accuracy of calculations using the most complete empirical formulae is about the same.

However, the use of formulae providing such high accuracy is quite cumbersome.

Therefore, a less but for most applications sufficient accurate formula is given by

$$
\begin{aligned}
c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 \\
+ (1.34 - 0.01T)(S - 35) + 0.016z
\end{aligned}
$$

with temperature T in (°C), salinity S in (ppt), depth z in (m) and sound velocity c in (m/s). This formula is valid for

$0\ ^\circ\mathrm{C} \le T \le 35\ ^\circ\mathrm{C}, \qquad 0\ \mathrm{ppt} \le S \le 45\ \mathrm{ppt}, \qquad 0\ \mathrm{m} \le z \le 1000\ \mathrm{m}.$

Assignment 1:

Develop a Matlab program for determining the sound speed

$$c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3$$
$$+(1.34 - 0.01T)(S - 35) + 0.016z.$$

as function of temperature T, salinity S and depth z.

Furthermore, to investigate the dependence of c on T, S and z visualize c versus z for various sets of T and S, i.e.

$$T = 5\ °\mathrm{C} : 5\ °\mathrm{C} : 30\ °\mathrm{C} \ \text{ and } \ S = 10\ \mathrm{ppt} : 5\ \mathrm{ppt} : 35\ \mathrm{ppt},$$

in appropriate diagrams and discuss the results obtained.

1.2 Typical Vertical Sound Velocity Profiles

The sound propagation conditions in the ocean are mainly determined by the shape of the sound velocity profile

$$c(z) = c(T(z), S(z), z),$$

i.e. its derivative (gradient)

$$\frac{d}{dz}c(z).$$

The sound velocity profiles $c(z)$

- are different in various ocean regions and
- vary with time, e.g. seasonal changes

The variation of T and S at depths below 1 km are usually fairly weak.

$\Rightarrow$ Sound velocity increases almost exclusively due to the increasing hydrostatic pressure, i.e. sound velocity increases linearly with depth.

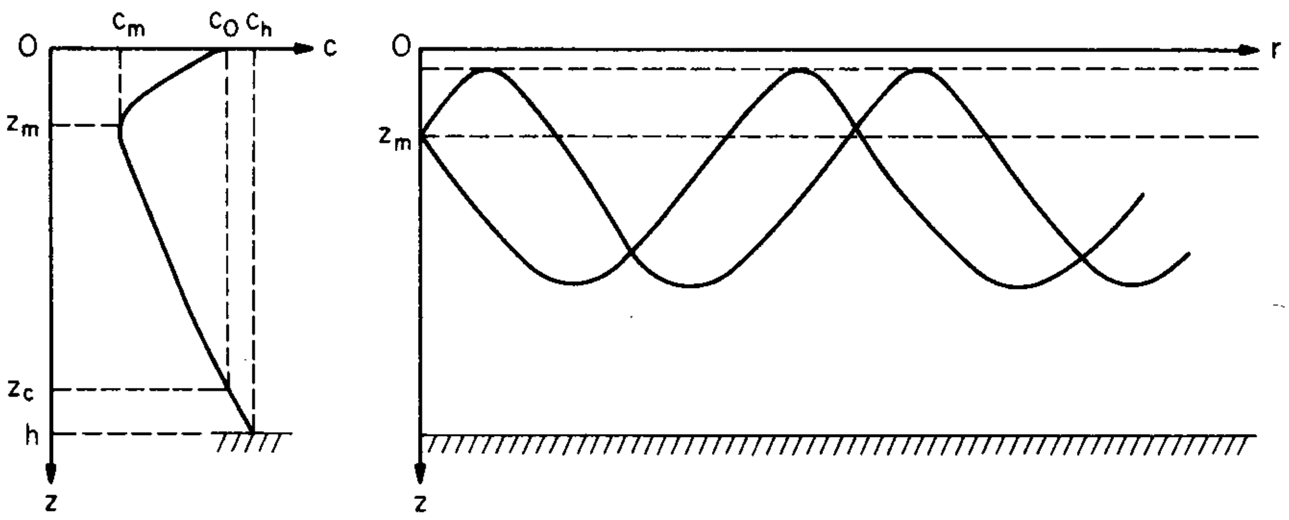
1.2.1 Underwater Sound Channel (USC)

Deep water regions, typical profiles possess a velocity minimum at a certain depth z_m, where

- z_m defines the axis of the Underwater sound channel
- above z_m the sound velocity increases mainly due to temperature increases
- below z_m the increase in hydrostatic pressure is mainly responsible for increasing the sound velocity

If a sound source is closely located to the axis of the USC the sound energy is partly trapped within the USC, i.e. some part of sound does not reach the bottom or surface and therefore does not undergo scattering and absorption at these boundaries.

Underwater Sound Channel of the First Kind, i.e. $c_0 < c_h$



Waveguide propagation can be observed in the domain

$$0 < z < z_c,$$

where the depths

$$z = 0 \quad \text{and} \quad z = z_c$$

define the boundaries of the USC.

The channel traps all sound rays that leave a source located on the USC axis at grazing angles

$$\varepsilon < \varepsilon_{\max} \quad \text{with} \quad \varepsilon_{\max} = \sqrt{\frac{2(c_0 - c_m)}{c_m}},$$

where c_m and c_0 are the sound velocities at the axis and the boundaries of the USC.

The greater the sound velocity difference

$$\Delta c = c_0 - c_m$$

the larger is the angular interval

$$[0, \varepsilon_{\max}]$$

in which the rays are trapped, i.e. the waveguide is getting more effective.

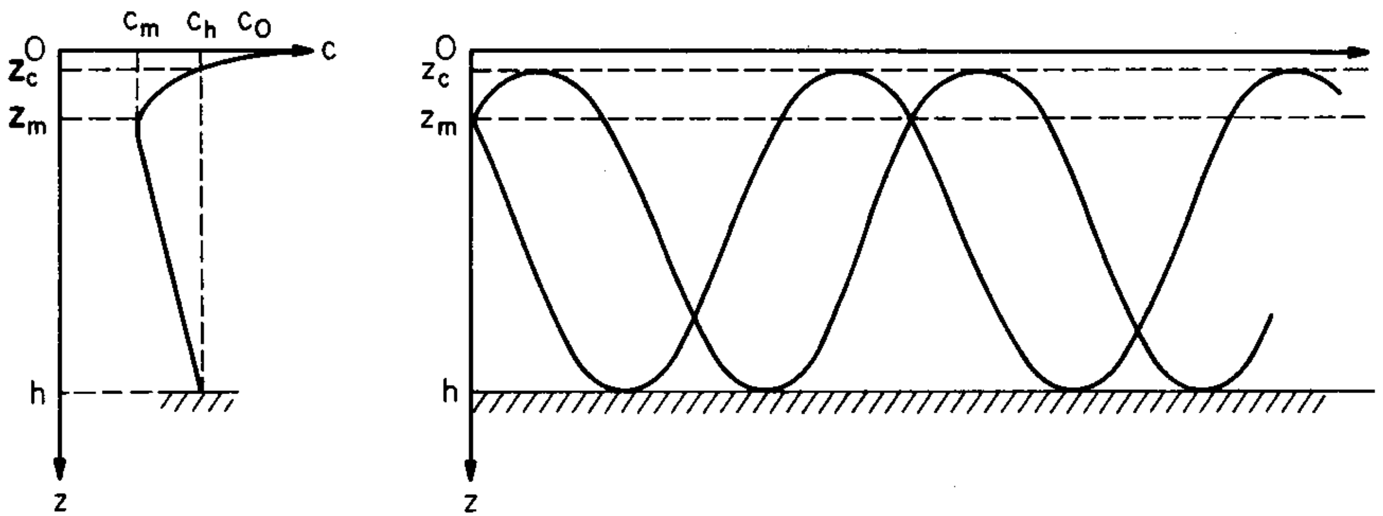
The depth z_m of the USC axis

- lies in a domain of 1000 m to 1200 m at mid-latitudes
- tends towards the ocean surface in polar regions
- can fall down to 2000 m in tropical areas

At moderate latitudes, i.e. 60° S to 60° N, the sound velocity c_m on the USC axis ranges from

- 1450 m/s to 1485 m/s in the Pacific Ocean
- 1450 m/s to 1500 m/s in the Atlantic Ocean

Underwater Sound Channel of the Second Kind, i.e. $c_0 > c_h$



The USC of the second kind extends from the bottom depth $z = h$ up to the depth $z = z_c$, where the sound speed equals c_h.

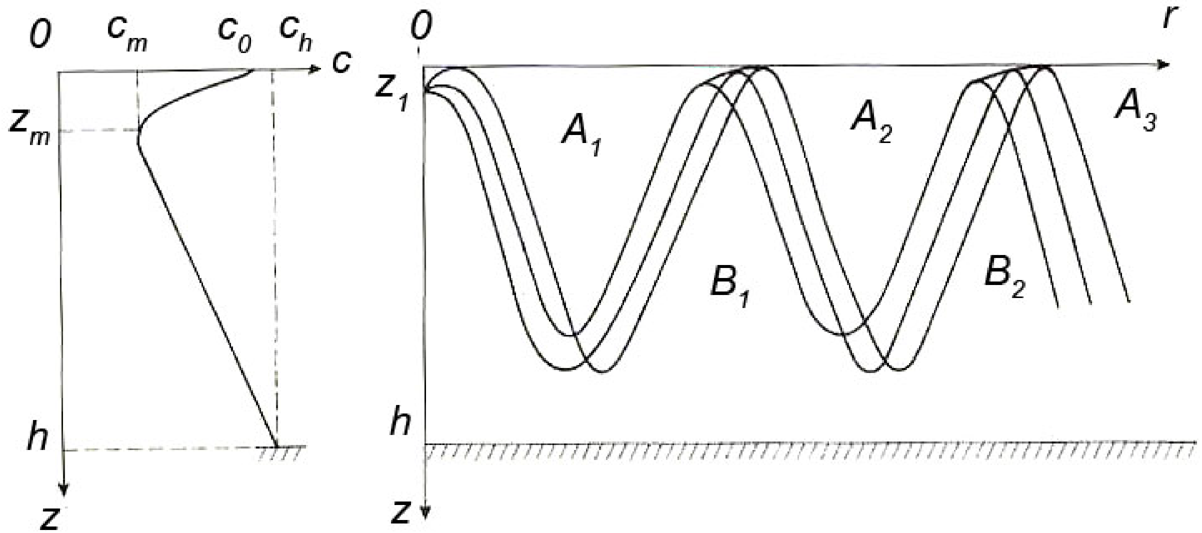
USC's of the first kind, i.e. $c_0 < c_h$, occur in deep water areas, whereas USC's of the second kind, i.e. $c_0 > c_h$, are more likely in regions of shallower water.

Typical Zonal Structure of a Sound Field in a USC

For sources near the ocean surface typical so-called

"zonal structures"

of the sound field, i.e. sequences of insonified and shadow zones, can be observed in the ray diagram



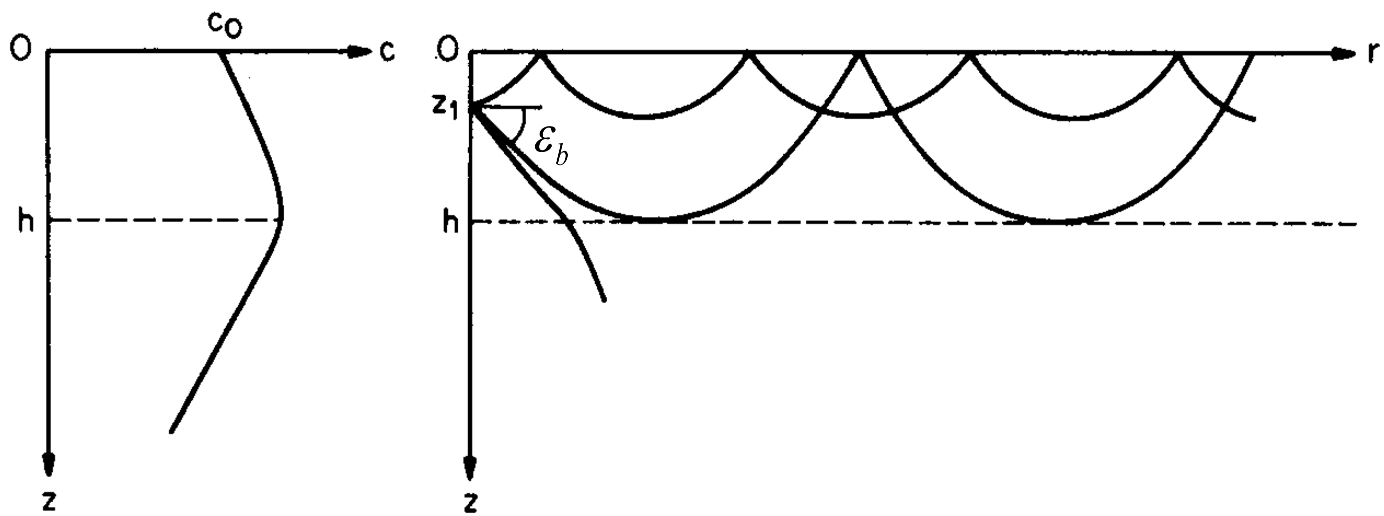
where $A_1, A_2, \ldots$ and $B_1, B_2, \ldots$ indicate the shadow zones.

The Shadow zones

- decrease as the source depth z_1 approaches the USC axis
- disappear if the source depth z_1 coincides with the depth of the USC axis z_m, i.e. $z_m = z_1$.

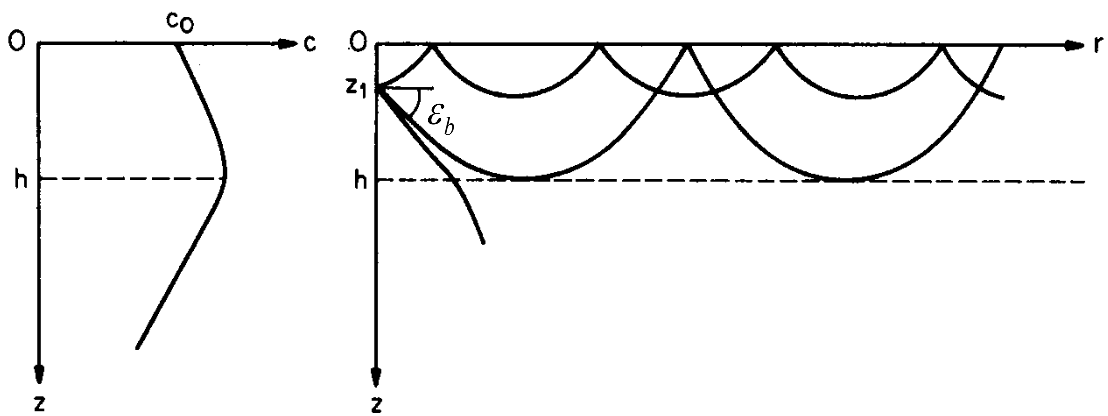
1.2.2 Surface Sound Channel

Such sound channels are formed when the channel axis coincides with the surface.



The sound velocity increases down to the depth $z = h$ and then begins to decrease.

Rays leaving the source at grazing angles $\varepsilon < \varepsilon_b$ propagate with multiple surface reflection in the surface sound channel.



In case of a rough ocean surface, sound is partly scattered into angles $\varepsilon > \varepsilon_b$ at each interaction with the surface, i.e.

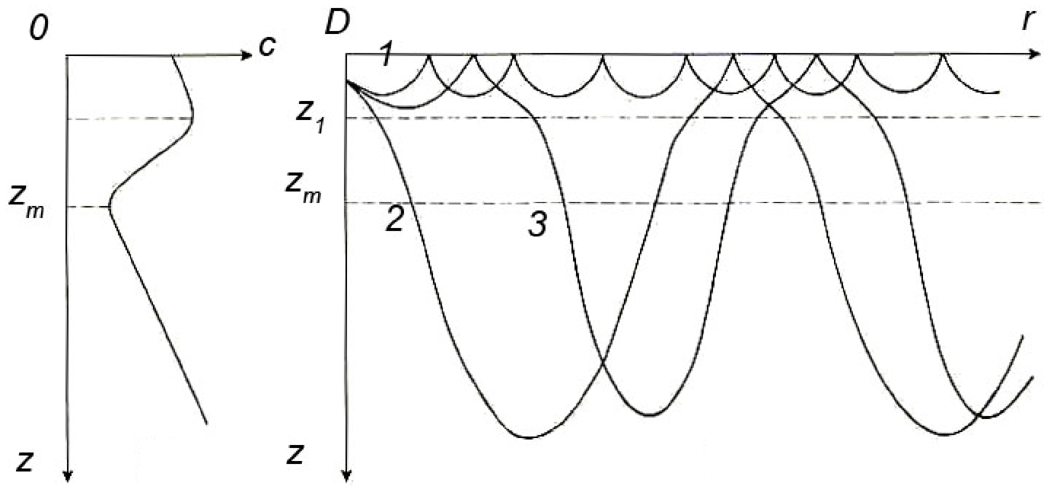
- rays leave the sound channel
- sound levels decay in the surface sound channel and increase below the surface sound channel.

Surface sound channels frequently occur

- in tropical and moderate zones of the ocean, where c increases with depth due to the positive hydrostatic pressure gradient when T and S are almost constant due to wind mixing in the upper ocean layer
- in Arctic and Antarctic regions, where c monotonically increases from the surface to the bottom due to the positive temperature and hydrostatic pressure gradient
- in Mediterranean seas, in the tropical zone and in shallow seas, where c increases with depth if the temperature on the surface decays due to changes during autumn and winter

1.2.3 Underwater Sound Channel with two Axis

This case takes place when surface and deep water sound channels are present simultaneously.



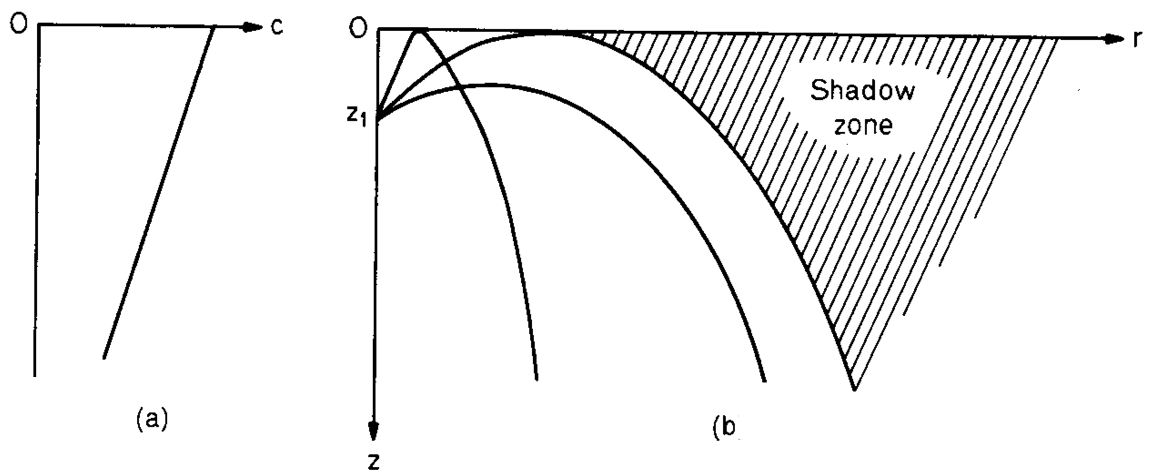
Such a distribution of the sound velocity with depth is due to the intrusion of warmer and saltier Mediterranean waters into the Atlantic Ocean, e.g. off the cost of Portugal.

1.2.4 Antiwaveguide Propagation

An antiwaveguide propagation is observed when the sound velocity monotonically decreases with depth.

Such sound velocity profiles are often a result of intensive heating by solar radiation of the upper ocean layer.

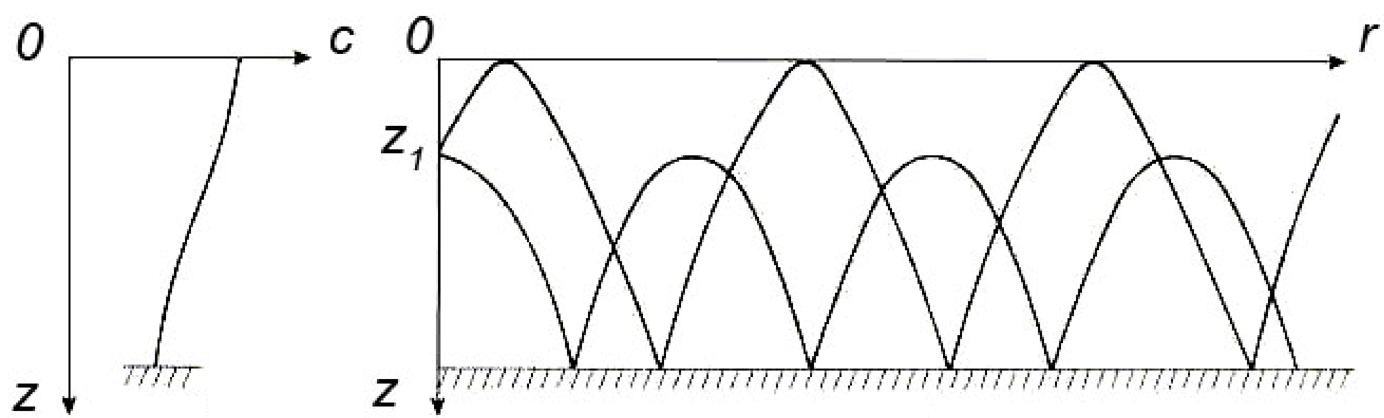
Formation of a geometrical shadow



The shadow zone is not a region of zero sound intensity since sound waves penetrate into the shadow zone due to

- diffraction
- reflections at sea floor
- scattering by inhomogeneities of the medium

Sound Propagation in Shallow Water



1.3 Transmission Loss of Sound

1.3.1 Spreading Loss

Spreading loss is a measure of signal weakening due to the geometrical spreading of a wave propagating outward from the source.

Two geometries are of importance in underwater acoustics

- spherical spreading, i.e. a point source in an unbounded homogeneous medium
- cylindrical spreading, i.e. a point source in a medium that has upper and lower boundaries (waveguide case)

If we assume the medium to be lossless the intensity is for

- spherical spreading inversely proportional to the surface of the sphere of radius R, i.e.

$$I \propto \frac{1}{4\pi R^2}$$

- cylindrical spreading inversely proportional to the surface of the cylinder of radius r and depth d, i.e.

$$I \propto \frac{1}{2\pi r d}$$

__Remark:__

For a point source in a homogeneous waveguide one observes

- spherical spreading in the near field, i.e. $R \leq d$
- cylindrical spreading at long range, i.e. $r >> d$
- transition region from spherical towards cylindrical spreading in-between

1.3.2 Sound Attenuation in Water

The acoustic energy of a sound wave propagating in the ocean is partly

- absorbed, i.e. the energy is transformed into heat
- lost due to sound scattering by inhomogeneities

Remark:

It is not possible to distinguish between absorption and scattering effects in real ocean experiments. Both phenomena contribute simultaneously to the sound attenuation in sea water.

On the basis of extensive laboratory and field experiments the following empirical formulae for the calculation of attenuation coefficients in sea water have been derived.

Thorp formula (valid for 100 Hz < f < 3 kHz)

$$\alpha_w = \frac{0.11 f^2}{1+f^2} + \frac{44 f^2}{4100+f^2} \quad [\text{dB/km}] \qquad \text{with} \quad f \ \text{ in } \ [\text{kHz}]$$

Schulkin and Marsh formula (valid for 3 kHz < f < 0.5 MHz)

$$\alpha_w = 8.686 \cdot 10^3 \left(\frac{S A f_T f^2}{f_T{}^2 + f^2} + \frac{B f^2}{f_T} \right) (1 - 6.54 \cdot 10^{-4} P) \quad [\text{dB/km}],$$

where

$$A = 2.34 \cdot 10^{-6}, \ \ B = 3.38 \cdot 10^{-6}, \ \ S \ \text{ in [ppt]}, \ \ f \ \text{ in [kHz]},$$

the relaxation frequency

$$f_T = 21.9 \cdot 10^{6-1520/(T+273)} \quad \text{with} \quad T \ \text{ in [°C]} \quad \text{for } \ 0 \le T \le 30\text{°C}$$

and the hydrostatic pressure is determined by

$$P = 1.01(1 + z \cdot 0.1) \quad \text{in} \quad [\text{kg/cm}^2 = \text{at}]$$

<u>Francois and Garrison Formula</u> (valid for 100 Hz < f < 1 MHz)

$$\alpha_w = \underbrace{\frac{A_1 P_1 f_1 f^2}{f_1^2 + f^2}}_{\substack{\text{B(OH)}_3 \\ \text{boric acid}}} + \underbrace{\frac{A_2 P_2 f_2 f^2}{f_2^2 + f^2}}_{\substack{\text{MgSO}_4 \\ \text{magnesium sulphate}}} + \underbrace{A_3 P_3 f^2}_{\substack{\text{pure water} \\ \text{viscosity}}} \quad [\text{dB/km}]$$

The coefficients for the contribution of

- boric acid, $B(OH)_3$

$$A_1 = \frac{8.686}{c} 10^{0.78\,ph - 5}, \quad f_1 = 2.8\sqrt{\frac{S}{35}} \cdot 10^{4 - \frac{1245}{T + 273}}$$

$$P_1 = 1, \quad c = 1412 + 3.21\,T + 1.19\,S + 0.0167\,z_{\max}$$

- magnesium sulphate, $MgSO_4$

$$A_2 = 21.44\,\frac{S}{c}\,(1+0.025T), \quad f_2 = \frac{8.17\cdot 10^{8-1990/(T+273)}}{1+0.0018\,(S-35)}$$

$$P_2 = 1 - 1.37\cdot 10^{-4} z_{\max} + 6.2\cdot 10^{-9}\cdot z_{\max}^2$$

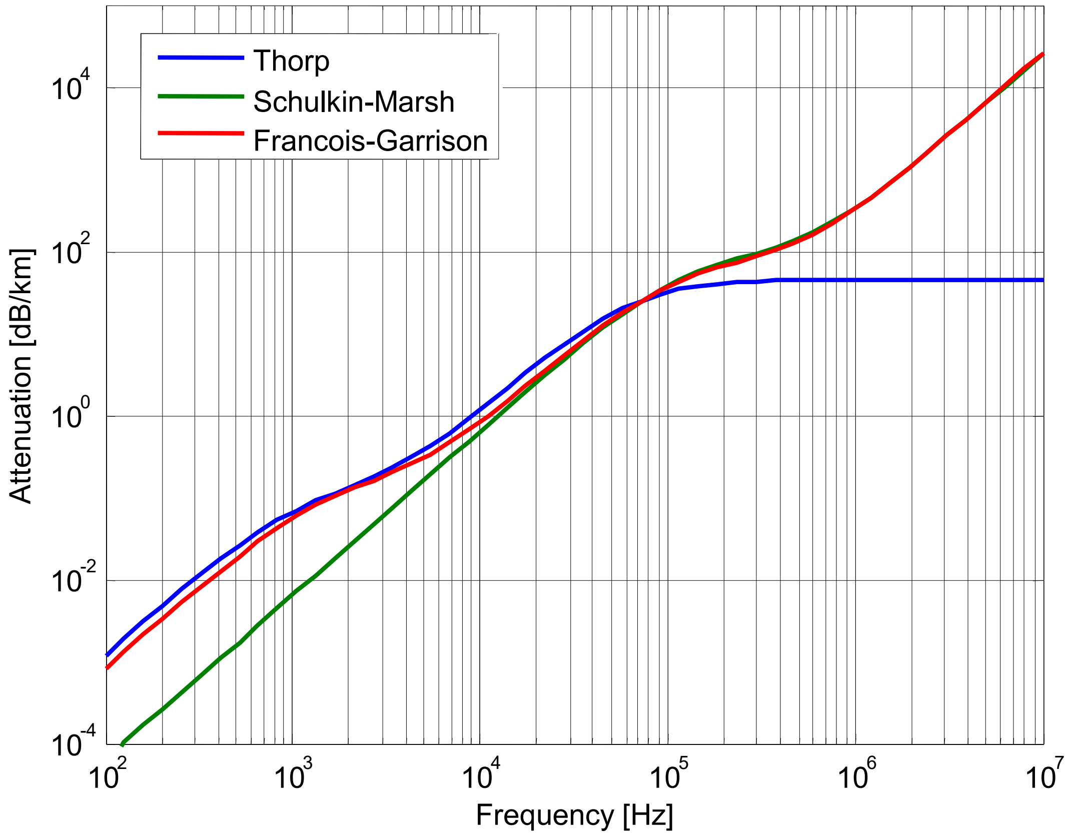
- pure water viscosity

$$A_3 = \begin{cases} 4.937\cdot 10^{-4} - 2.59\cdot 10^{-5}T + 9.11\cdot 10^{-7}T^2 - 1.5\cdot 10^{-8}T^3 & \text{for } T \le 20\,°\text{C} \\ 3.964\cdot 10^{-4} - 1.146\cdot 10^{-5}T + 1.45\cdot 10^{-7}T^2 - 6.5\cdot 10^{-10}T^3 & \text{for } T \ge 20\,°\text{C} \end{cases}$$

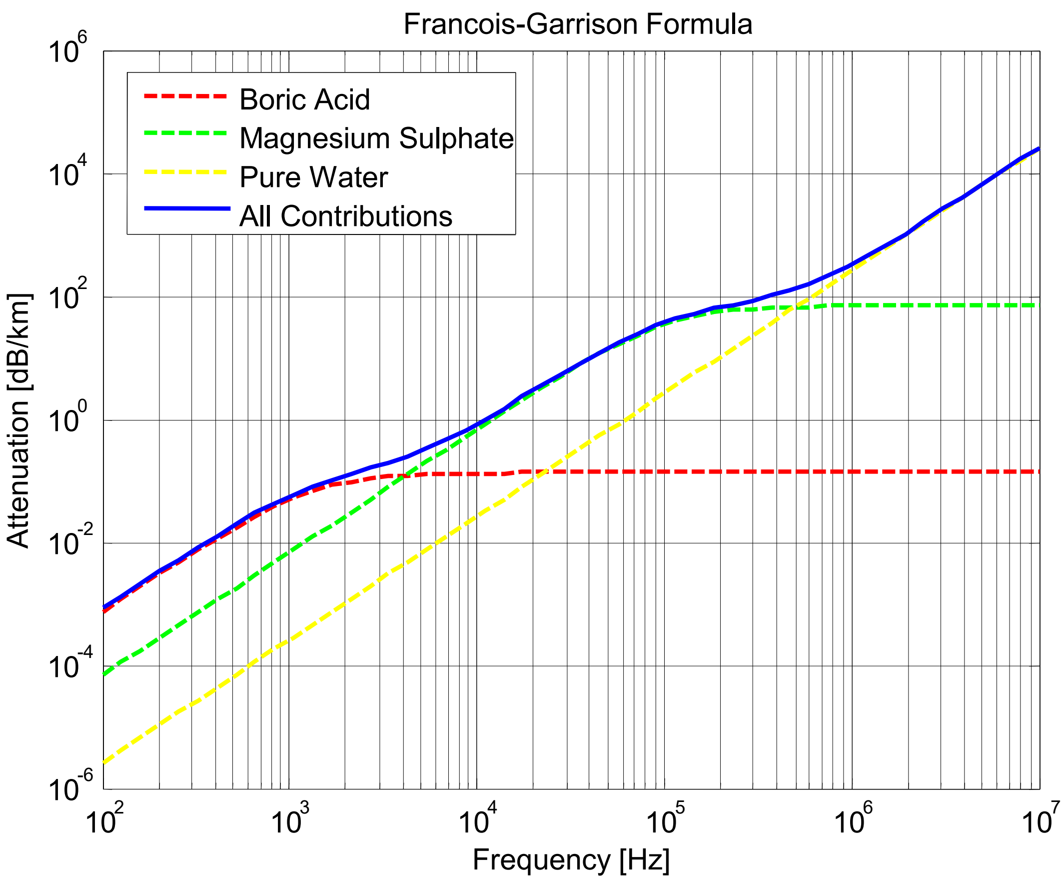
$$P_3 = 1 - 3.83\cdot 10^{-5} z_{\max} + 4.9\cdot 10^{-10} z_{\max}^2$$

with f in [kHz], T in [˚C], S in [ppt]. Furthermore $z_{\max}$, ph and c denote the water depth in [m], the *ph*-value and the sound speed in [m/s] respect-tively.

Comparison of the Thorp, Schulkin-Marsh and Francois-Garrison attenuation formulae.



Frequency dependence of the different attenuation processes employed in the Francois-Garrison model.



Assignment 2:

Develop a Matlab program that calculates the sound attenuation in seawater by means of the

- Thorp formula
- Schulkin & Marsh formula
- Francois & Garrison formula.

Display and compare the results of the three approaches.

For the Francois and Garrison formula

- investigate the dependence on the frequency f, salinity S and temperature T for a depth of $z = 50$ m
- depict the attenuation versus frequency for a particular S, T and z and specify the frequency regions where the different attenuation processes dominate.

1.3.3 Sound Attenuation in Sediment

The sound attenuation in the sediment mainly varies with the bottom type. It can be approximately determined by the empirical formula

$$\alpha_s = \frac{1}{8.686} K \left(\frac{f}{1\ \mathrm{kHz}} \right)^n \quad \left[\frac{1}{\mathrm{m}} \right],$$

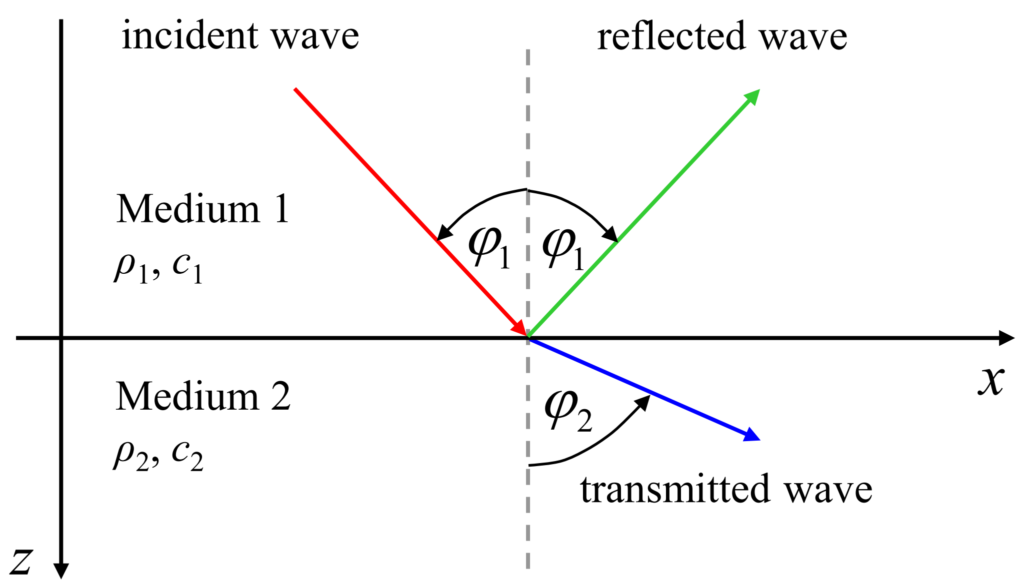
where K and n denote two bottom type dependent parameters. The following table provides the values for K and n for four representative sediment types.

Parameter	**Sediment type**			
	very fine silt	fine sand	medium sand	coarse sand
K	0.17	0.45	0.48	0.53
n	0.96	1.02	0.98	0.96

1.4 Sound Reflection and Transmission at a fluid-fluid interface

1.4.1 Lossless Media

Reflectivity is the ratio of the amplitudes of a reflected plane wave to a plane wave incident on an inter-face separating two media. It is an impor-tant measure for the impact of the sea sur-face and bottom on sound propagation.



Assuming the incident wave to have amplitude A and denoting the reflection and transmission coefficients by R and T, respectively, we can write

$$p_i = A\exp\left\{j(\omega t - \mathbf{k}_i^T\mathbf{r})\right\} \qquad \mathbf{k}_i^T = k_1(\sin\varphi_1, \cos\varphi_1)$$

$$p_r = R\,A\exp\left\{j(\omega t - \mathbf{k}_r^T\mathbf{r})\right\} \qquad \mathbf{k}_r^T = k_1(\sin\varphi_1, -\cos\varphi_1)$$

$$p_t = T\,A\exp\left\{j(\omega t - \mathbf{k}_t^T\mathbf{r})\right\} \qquad \mathbf{k}_t^T = k_2(\sin\varphi_2, \cos\varphi_2)$$

with

$$k_1 = \frac{\omega}{c_1}, \quad k_2 = \frac{\omega}{c_2}, \quad \mathbf{r} = (x, z)^T.$$

The unknown quantities R, T and φ_2 are determined from the boundary conditions requiring continuity of pressure and vertical particle velocity across the interface at $z = 0$.

The boundary conditions can be mathematically stated as

$$p_1 = p_i + p_r = p_t = p_2 \quad \text{and} \quad \frac{\partial v_{z,1}}{\partial t} = -\frac{1}{\rho_1}\frac{\partial p_1}{\partial z} = -\frac{1}{\rho_2}\frac{\partial p_2}{\partial z} = \frac{\partial v_{z,2}}{\partial t},$$

respectively. After substituting p_i, p_r and p_t in the first boundary condition, we obtain

$$1 + R = T\exp\{j(k_1\sin\varphi_1 - k_2\sin\varphi_2)x\}.$$

Since the left side is independent of x, the right side must also be independent of x, i.e.

$$k_1\sin\varphi_1 - k_2\sin\varphi_2 = 0.$$

This leads to $1 + R = T$ and to the well-known refraction law

$$\frac{\sin\varphi_1}{\sin\varphi_2} = \frac{k_2}{k_1} = \frac{\omega/c_2}{\omega/c_1} = \frac{c_1}{c_2} = n.$$

The second boundary condition and $k_1 \sin\varphi_1 = k_2 \sin\varphi_2$ provide

$$\frac{k_1}{\rho_1}(1-R)\cos\varphi_1 = \frac{k_2}{\rho_2} T \cos\varphi_2 .$$

Defining $m = \rho_2/\rho_1$ and using the last equation together with $1+R=T$ and $n = k_2/k_1$ we find

$$R = \frac{m\cos\varphi_1 - n\cos\varphi_2}{m\cos\varphi_1 + n\cos\varphi_2} = \frac{m\cos\varphi_1 - \sqrt{n^2 - \sin^2\varphi_1}}{m\cos\varphi_1 + \sqrt{n^2 - \sin^2\varphi_1}}$$

and

$$T = \frac{2m\cos\varphi_1}{m\cos\varphi_1 + n\cos\varphi_2} = \frac{2m\cos\varphi_1}{m\cos\varphi_1 + \sqrt{n^2 - \sin^2\varphi_1}},$$

where $n\cos\varphi_2 = \sqrt{n^2 - \sin^2\varphi_1}$ for $n \geq \sin\varphi_1 \geq 0$ has been exploited.

Remarks:

Features of the reflection and transmission coefficient

a) If φ_1 tends to $\pi/2$ then R and T tend independently of the parameters of the media to -1 and 0, respectively.

b) At the angle of incidence φ_1 that satisfies

$$\sin\varphi_1 = \sqrt{\frac{m^2 - n^2}{m^2 - 1}}, \quad \text{i.e. } R = 0$$

the boundary will be completely transparent.

c) For $\sin\varphi_1 > n$ and $n\cos\varphi_2 = \pm j\sqrt{\sin^2\varphi_1 - n^2}$ the reflection coefficient can be expressed by

$$R = \frac{m\cos\varphi_1 + j\sqrt{\sin^2\varphi_1 - n^2}}{m\cos\varphi_1 - j\sqrt{\sin^2\varphi_1 - n^2}} \quad \left(\begin{array}{l}\text{finiteness of refracted wave re-}\\ \text{quires negative sign of the root}\end{array}\right)$$

or after some manipulations by

$$R = \exp(j\vartheta) \quad \text{with} \quad |R| = 1 \ \text{ and } \ \vartheta = 2\arctan\left(\frac{\sqrt{\sin^2\varphi_1 - n^2}}{m\cos\varphi_1}\right),$$

i.e. total reflection occurs.

The phase difference between the incident and reflected waves at the interface is given by ϑ.

The angle of incidence satisfying $\sin\varphi_1 = n$ is called critical angle φ_{crit}.

For $\varphi_1 > \varphi_{crit}$ and $k_2\cos\varphi_2 = k_1 n\cos\varphi_2 = \pm j k_1\sqrt{\sin^2\varphi_1 - n^2}$ the amplitude of the transmitted sound pressure satisfies

$$|p_t| \propto \exp(-\delta z) \quad \text{with} \quad \delta = k_1\sqrt{\sin^2\varphi_1 - n^2}.$$

(finiteness of refracted wave requires negative sign of the root)

Example: Water-Air / Air-Water Interface

With the densities and sound velocities of water and air

$$\rho_w = 1\frac{\mathrm{g}}{\mathrm{cm}^3} = 1000\frac{\mathrm{kg}}{\mathrm{m}^3}, \quad c_w = 1500\frac{\mathrm{m}}{\mathrm{s}}$$

$$\rho_a = 1.3\cdot 10^{-3}\frac{\mathrm{g}}{\mathrm{cm}^3} = 1.3\frac{\mathrm{kg}}{\mathrm{m}^3}, \quad c_a = 333\frac{\mathrm{m}}{\mathrm{s}},$$

we obtain for a sound wave impinging perpendicular from water into air and vice versa the reflection and transmission coefficients

$$R \approx -1, \quad T \approx 0 \qquad \text{and} \qquad R \approx 1, \quad T \approx 2,$$

respectively.

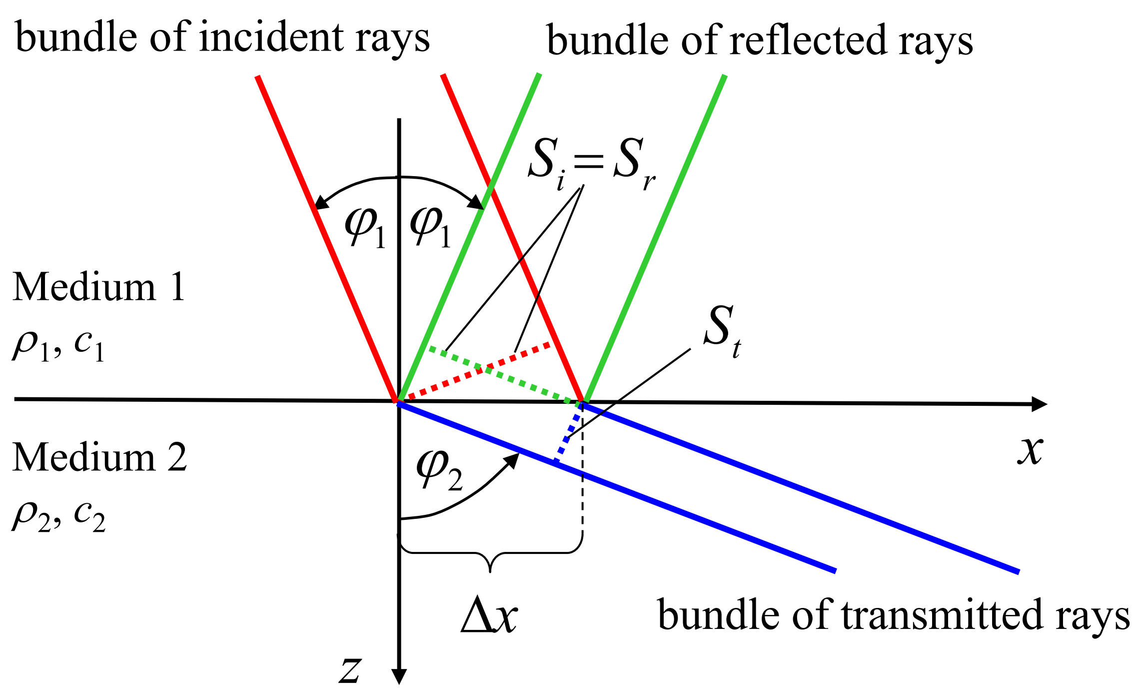
The intensities (power flux densities) of the incident, reflected and transmitted waves are defined by

$$I_i = \frac{|p_i|^2}{2\rho_1 c_1} = \frac{A^2}{2\rho_1 c_1}, \qquad I_r = \frac{|p_r|^2}{2\rho_1 c_1} = \frac{R^2 |p_i|^2}{2\rho_1 c_1} = \frac{R^2 A^2}{2\rho_1 c_1}$$

and

$$I_t = \frac{|p_t|^2}{2\rho_2 c_2} = \frac{T^2 |p_i|^2}{2\rho_2 c_2} = \frac{T^2 A^2}{2\rho_2 c_2}.$$

Moreover, to derive the power reflection and transmission coefficients R_P and T_P, one has to take into account that the refraction at the interface changes the intensity because of a change in the cross-sectional area, cf. following figure.



The cross-sections of the incident, reflected and transmitted bundle of rays are given by

$$S_i = S_r = l_y \Delta x \cos \varphi_1 \quad \text{and} \quad S_t = l_y \Delta x \cos \varphi_2 .$$

Using these expressions, we obtain

$$R_P = \frac{P_r}{P_i} = \frac{S_r I_r}{S_i I_i} = \frac{I_r}{I_i} = R^2 = \left(\frac{m\cos\varphi_1 - n\cos\varphi_2}{m\cos\varphi_1 + n\cos\varphi_2} \right)^2$$

and

$$T_P = \frac{P_t}{P_i} = \frac{S_t I_t}{S_i I_i} = \frac{\cos\varphi_2}{\cos\varphi_1} \cdot \frac{I_t}{I_i} = \frac{\cos\varphi_2}{\cos\varphi_1} \cdot \frac{\rho_1 c_1}{\rho_2 c_2} T^2$$

$$= \frac{\cos\varphi_2}{\cos\varphi_1} \cdot \frac{n}{m} \cdot \frac{4m^2\cos^2\varphi_1}{\left(m\cos\varphi_1 + n\cos\varphi_2\right)^2} = \frac{4nm\cos\varphi_1\cos\varphi_2}{\left(m\cos\varphi_1 + n\cos\varphi_2\right)^2}.$$

One can now prove that the law of energy conservation, i.e. $P_i = P_r + P_t$ implying $R_P + T_P = 1$ is satisfied. Furthermore, it can be shown that R_P and T_P remain unchanged if we reverse the direction of propagation, i.e.

$$\rho_1 \rightleftarrows \rho_2, c_1 \rightleftarrows c_2, \varphi_1 \rightleftarrows \varphi_2.$$

1.4.2 Lossy Media

In the former section the reflection and transmission properties of sound at plan boundaries have been deduced if the absorption of the media can be neglected.

In contrast to sound propagation at the water-air-boundary-layer experimental investigations at the water-sediment-boundary-layer show that the theory agrees sufficiently exact with the results of measurements only if the absorption in the sediment is taken into account.

Therefore, the results of Section 1.4.1 are now extended for the case of a boundary layer between a absorption-free medium 1 (water) and an absorption-afflicted medium 2 (sediment).

The absorption is introduced by means of the complex wave number

$$k_2 = k_{2,R} + j\,k_{2,I}$$

with

$$k_{2,R} = \frac{\omega}{c_2} \quad \text{and} \quad k_{2,I} = -\alpha_2,$$

where c_2 and α_2 denote the velocity of sound and attenuation of the medium 2 (sediment) respectively. The attenuation of medium 1 (water) can be neglected.

Thus, the refraction law can be written as

$$\frac{\sin\varphi_1}{\sin\varphi_2} = \frac{k_2}{k_1} = \frac{k_{2,R}}{k_1} + j\,\frac{k_{2,I}}{k_1} = n_R + j\,n_I = n.$$

The wave number k_1 and the incidence angle φ_1 are always real. Thus, the product $k_1 \sin\varphi_1$ is also real. Since, the wave number k_2 is complex and $k_2 \sin\varphi_2$ has to be real due to the refraction law,

$$\sin\varphi_2 = \frac{k_1 \sin\varphi_1}{k_2}$$

has to be complex. Hence,

$$k_2 \cos\varphi_2 = k_2\sqrt{1-\sin^2\varphi_2} = k_2\sqrt{1-k_1^2 \sin^2\varphi_1 / k_2^2}$$
$$= k_1\sqrt{n^2 - \sin^2\varphi_1} = \tilde{k}_{2,R} + j\,\tilde{k}_{2,I}$$

and

$$k_1 \sin\varphi_1 = k_2 \sin\varphi_2 .$$

The transmitted sound pressure can now be expressed by

$$p_t = T\,A\exp\left\{(0,\tilde{k}_{2,I})\mathbf{r}\right\}\exp\left\{j\left(\omega t-(k_1\sin\varphi_1,\tilde{k}_{2,R})\mathbf{r}\right)\right\}$$

$$= T\,A\exp\left\{\mathbf{k}_A^T\mathbf{r}\right\}\exp\left\{j(\omega t-\mathbf{k}_P^T\mathbf{r})\right\},$$

where

$$\mathbf{k}_P = (k_1\sin\varphi_1,\tilde{k}_{2,R})^T = k_P(\sin\varphi_{2,P},\cos\varphi_{2,P})^T$$

$$\mathbf{k}_A = (0,\tilde{k}_{2,I})^T = k_A(\sin\varphi_{2,A},\cos\varphi_{2,A})^T$$

with

$$k_P = \sqrt{(k_1\sin\varphi_1)^2+(\tilde{k}_{2,R})^2} = k_1\sqrt{\sin^2\varphi_1+\left(\mathrm{Re}\left\{\sqrt{n^2-\sin^2\varphi_1}\right\}\right)^2}$$

and

$$k_A = \tilde{k}_{2,I} = k_1\,\mathrm{Im}\left\{\sqrt{n^2-\sin^2\varphi_1}\right\}.$$

The angles of refraction for the wave fronts of constant phase and constant amplitude are given by

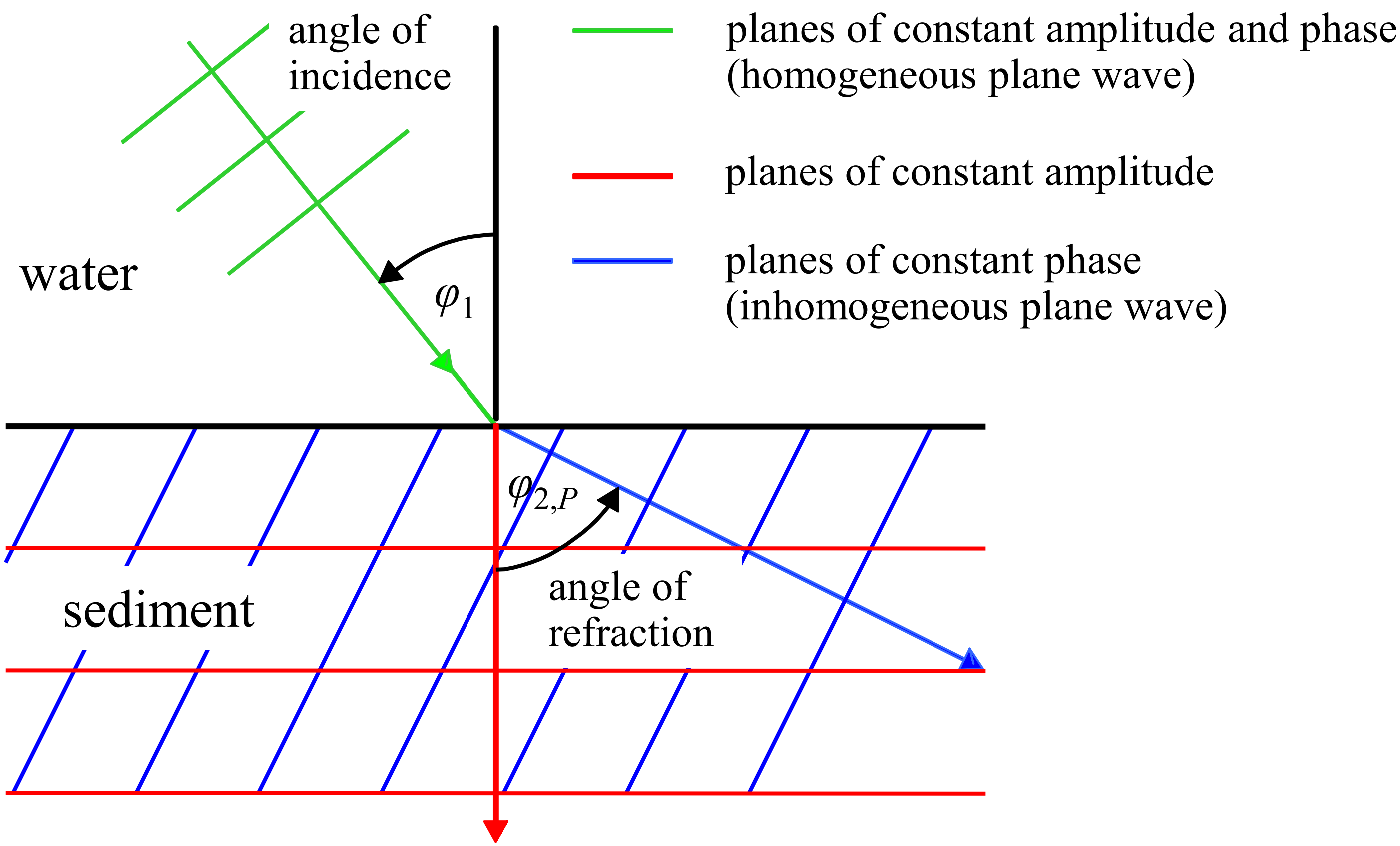
$$\varphi_{2,P} = \arg\left(\mathbf{k}_P\right) = \arctan\left(\frac{k_1 \sin\varphi_1}{\tilde{k}_{2,R}}\right) = \arctan\left(\frac{\sin\varphi_1}{\mathrm{Re}\left\{\sqrt{n^2 - \sin^2\varphi_1}\right\}}\right)$$

and

$$\varphi_{2,A} = \arg\left(\mathbf{k}_A\right) = 0,$$

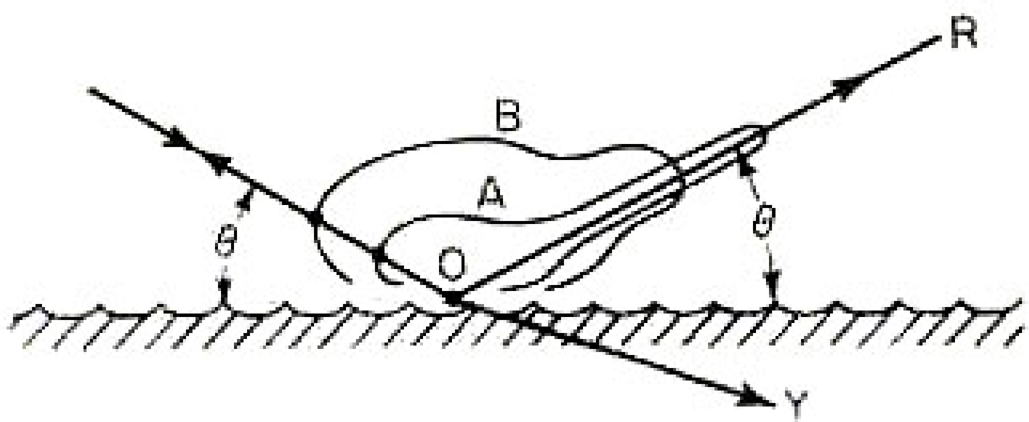
respectively. Furthermore, the phase velocity of the wave in the sediment can be written as

$$c_P = \frac{\omega}{k_P} = \frac{c_1}{\sqrt{\sin^2\varphi_1 + \left(\mathrm{Re}\left\{\sqrt{n^2 - \sin^2\varphi_1}\right\}\right)^2}}$$


angle of incidence
planes of constant amplitude and phase (homogeneous plane wave)
planes of constant amplitude
planes of constant phase (inhomogeneous plane wave)
water
φ_1
$\varphi_{2,P}$
sediment
angle of refraction

1.4.3 Forward Reflection Loss

A rough sea surface or seafloor causes attenuation of the acoustic field propagating in the ocean waveguide. The attenuation increases with increasing frequency. The field is scattered away from the specular direction.



Scattering characteristic for
A: smooth bottoms B: rough bottoms

The forward reflection loss due to a rough boundary is often simply modeled by incorporating an additional loss factor into the calculation of the specular reflection coefficient. A formula often used to describe reflectivity from a boundary is

$$\tilde{R}(\varphi) = R(\varphi)\, e^{-p^2/2},$$

where

$$p(\varphi) = 2\,k\,\sigma\cos\varphi$$

denotes the so-called Rayleigh parameter,

$$k = \frac{2\pi}{\lambda} \quad \text{with} \quad \lambda = \frac{c}{f}$$

the wavenumber, σ the RMS (root mean square) roughness and φ the angle of incidence.

The roughness of the ocean surface caused by wind induced waves is often modeled by the Pierson-Moskowitz or the Pierson-Neumann spectrum. The RMS roughness (wave height) of a fully developed wind wavefield can be approximately determined by

$$\sigma_{PM} \cong \sqrt{1.4 \cdot 10^{-5} v_w^4} \qquad \text{(Pierson-Moskowitz)}$$

$$\sigma_{PN} \cong \sqrt{0.341 \cdot 10^{-5} v_w^5} \qquad \text{(Pierson-Neumann),}$$

where v_w denotes the wind speed in [kn] (1 kn = 0.514 m/s).

The RMS roughness σ of an ocean seafloor is related to the mean grain size of the sediment. The following table provides the values of mean grain size and RMS roughness for various sediment types.

Sediment type	Mean Grain Size $[\varphi = -\log_2(a)]$	RMS Roughness σ [cm]
sandy gravel	−1	2.5
very coarse sand	− 0.5	2.25
coarse sand	0.5	1.85
medium sand	1.5	1.45
fine sand	2.5	1.15
very fine sand	3.5	0.85
coarse silt	4.5	0.7
medium silt	5.5	0.65
fine silt	6.5	0.6
very fine silt	7.5	0.55
silty clay	8.0	0.5
clay	9.0	0.5

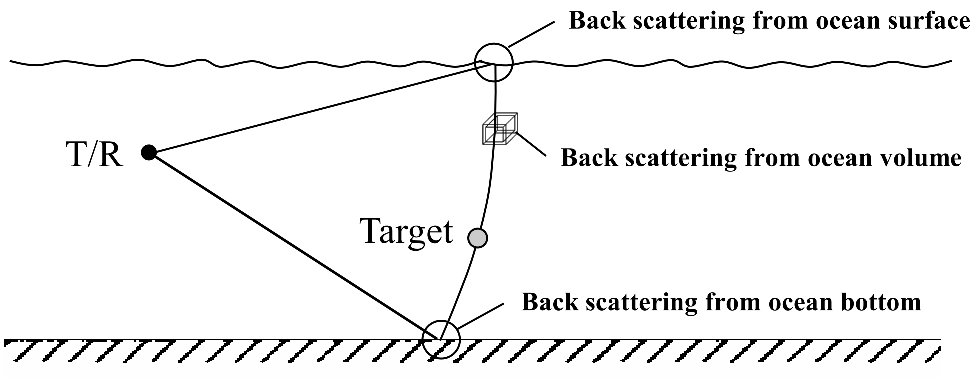
1.5 Sound Scattering

The sea contains, within itself and on its boundaries, inhomogeneities of many different kinds. These inhomogeneities reradiate a portion of the acoustic energy incident upon them. This reradiation of sound is called scattering. The total sum of all scattering contributions is called reverberation.

The reverberation basically produced by scatterers

- in the ocean volume (marine life, inanimate matter),
- on or near the ocean surface (roughness, air bubbles),
- on the ocean bottom (roughness)

is called volume reverberation, surface reverberation and bottom reverberation, respectively.



Surface Backscattering

Because of its roughness and the occurrence of air bubbles beneath it, the sea surface is a significant scatterer of sound. Experiments indicate that the backscattering strength of the sea surface varies with the

- grazing angle ($\theta = \pi/2 - \varphi$ with φ = angle of incidence),
- sound frequency and
- wind speed induced roughness,

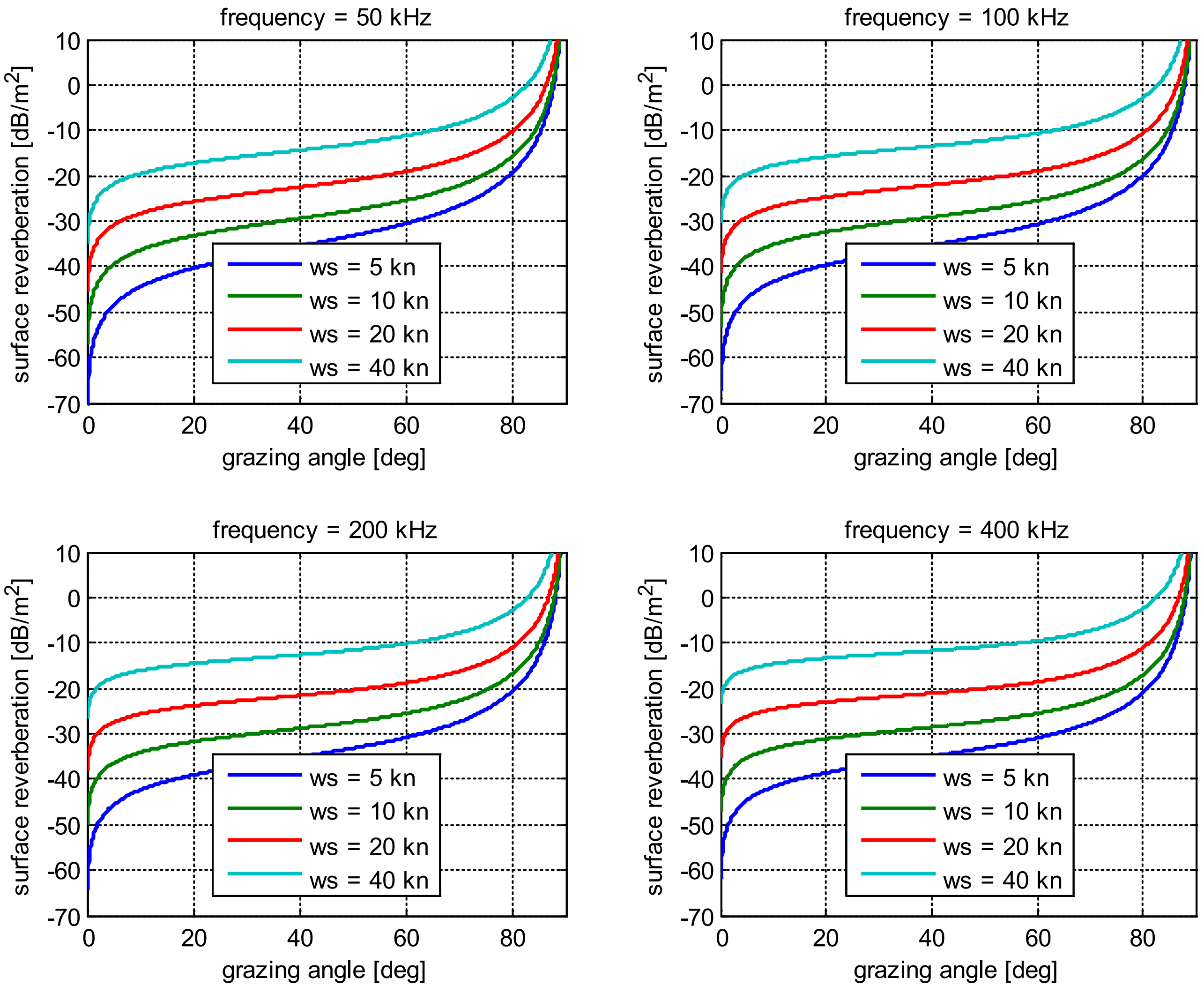
and that the collected measurements can be fitted by the following empirical expression

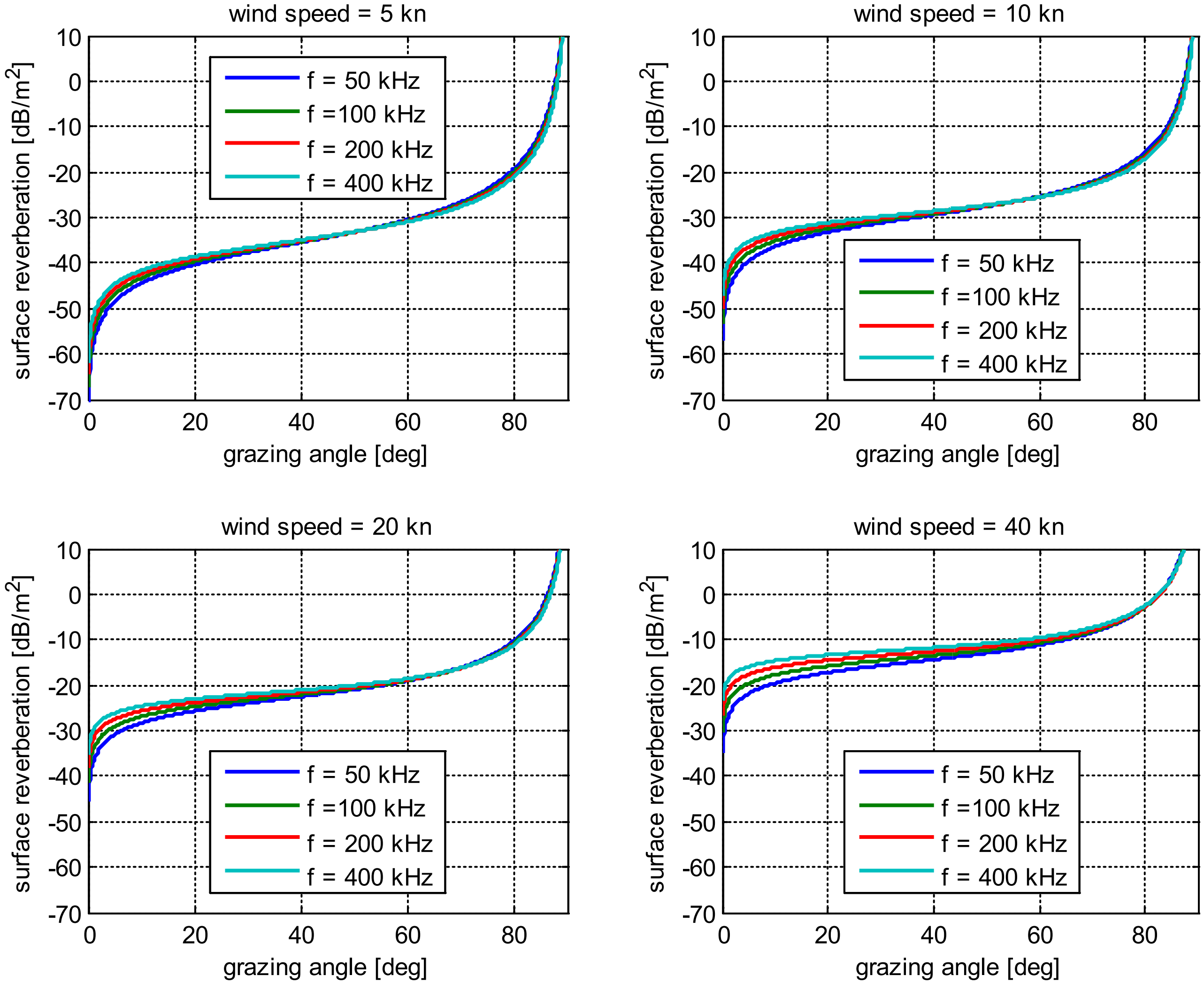
$$S_s = 10 \cdot \log_{10}\left(10^{-5.05} \cdot (1+v_w)^2 \cdot (f+0.1)^{v_w/150} \cdot \tan^{\beta}(\theta)\right) \quad [\mathrm{dB/m^2}]$$

with

$$\beta = 4 \cdot \left(\frac{v_w + 2}{v_w + 1}\right) + \left(2.5 \cdot (f+0.1)^{-1/3} - 4\right) \cdot \cos^{1/8}(\theta),$$

where S_S represents the surface backscattering coefficient in [dB/m²]. The parameters f, v_w and θ denote the sound frequency in kHz, the wind speed in knots and the grazing angle, respectively.


frequency = 50 kHz
frequency = 100 kHz
frequency = 200 kHz
frequency = 400 kHz
surface reverberation [dB/m2]
grazing angle [deg]
ws = 5 kn
ws = 10 kn
ws = 20 kn
ws = 40 kn
10
0
-10
-20
-30
-40
-50
-60
-70
0
20
40
60
80


wind speed = 5 kn
wind speed = 10 kn
wind speed = 20 kn
wind speed = 40 kn
surface reverberation [dB/m²]
grazing angle [deg]
f = 50 kHz
f =100 kHz
f = 200 kHz
f = 400 kHz
10
0
-10
-20
-30
-40
-50
-60
-70
0
20
40
60
80

Bottom Backscattering

The bottom acts, like the sea surface, due to its roughness as an reflector and scatterer of sound, cf. figure on p. 46.

In Section 1.4.3 the specular direction has been considered as part of the sound propagation via the forward reflection loss.

Now, we are going to model the backscattering behavior of the seabed. Experimental investigations have shown that the back-scattering strength of the bottom varies with the

- grazing angle ($\theta = \pi/2 - \varphi$ with φ = angle of incidence),
- sound frequency and
- bottom type induced roughness.

Furthermore, it could be observed that a Lambert's law relationship between the backscattering strength and the grazing angle fits to many experimental data satisfactorily accurate for angles below 60°.

Consequently, the backscattering strength can be described by Lambert's law and an empirically specified scattering coefficient, i.e.

$$S_B = K(f,bt) + 10 \cdot \log_{10}\left(\sin^2(\theta)\right) \quad [\mathrm{dB/m^2}], \quad 0 \le \theta \le 60°,$$

where $K(f,bt)$ denotes the scattering coefficient depending on the frequency of sound f and bottom type bt.

Due to the empirical definition of $K(f,bt)$ it is evident that one will have considerable difficulty in determining an appropriate value for the backscattering strength in practice.

Therefore, easier applicable and over the entire grazing angle domain sufficient accurate bottom backscattering models are of interest.

More accurate bottom scattering curves have been derived from measurements

- SEARAY Model (20 kHz $\leq f \leq$ 500 kHz)
- APL-UW Model (1 kHz $\leq f \leq$ 500 kHz)

For the SEARAY model the so-called reverberation coefficient is defined by

$$S_B = 10 \cdot \log_{10}\left(3.03 \cdot \beta \cdot f^{3.2-0.8 \cdot bt} 10^{2.8 \cdot bt - 12} + 10^{-4.42}\right) \quad [\text{dB/m}^2]$$

with

$$\beta = \gamma \cdot \left(\sin(\theta) + 0{,}19\right)^{bt \cdot \cos^{16}(\theta)}$$

and

$$\gamma = 1 + 125 \cdot \exp\left(-2.64 \cdot (bt - 1.75)^2 - \frac{50}{bt} \cdot \cot^2(\theta) \right),$$

where f, bt and θ denote the sound frequency in kHz, the bottom type and the grazing angle respectively.

The bottom type parameter is defined as follows

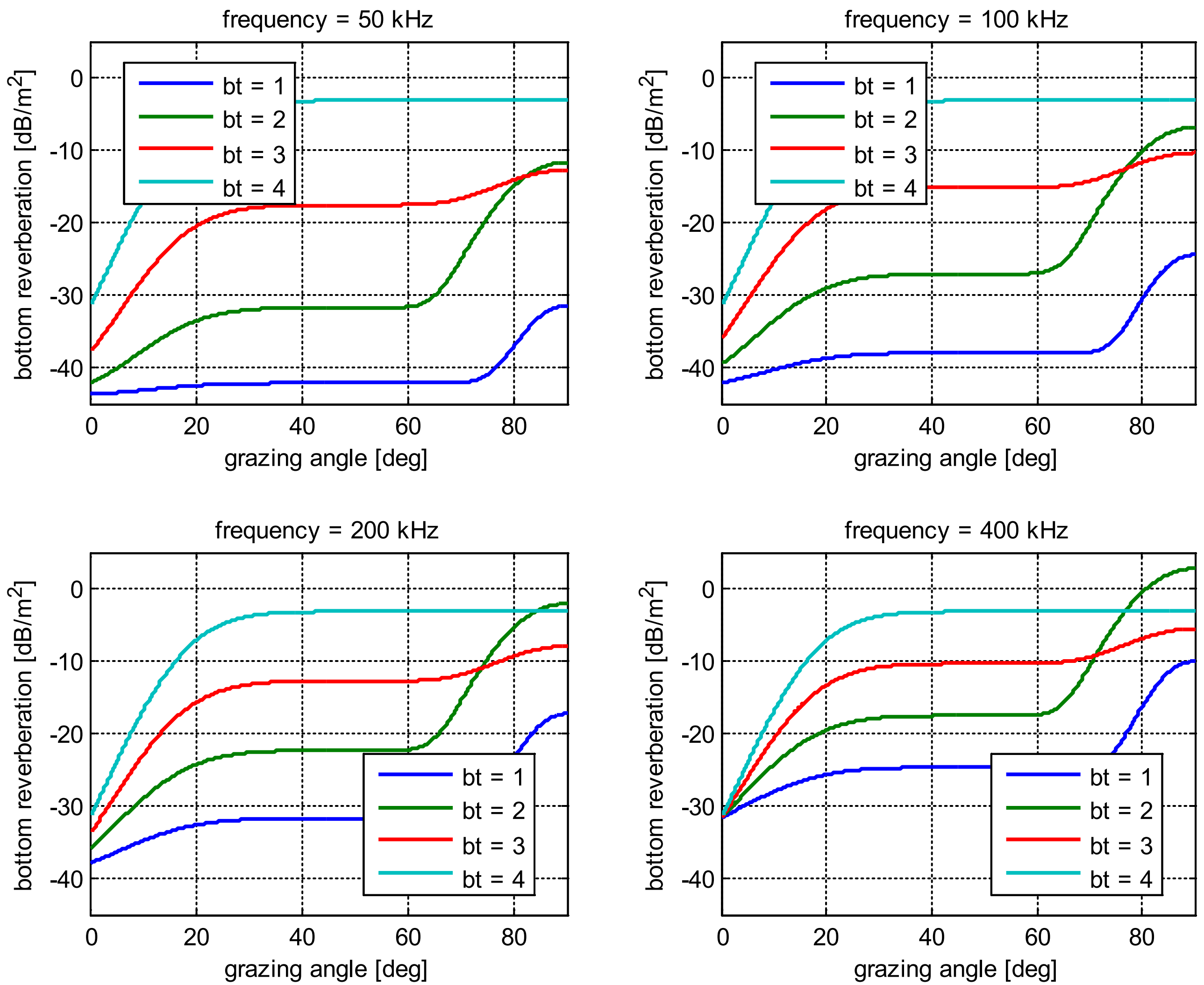
$bt = 1$ mud $\quad$ $bt = 2$ sand

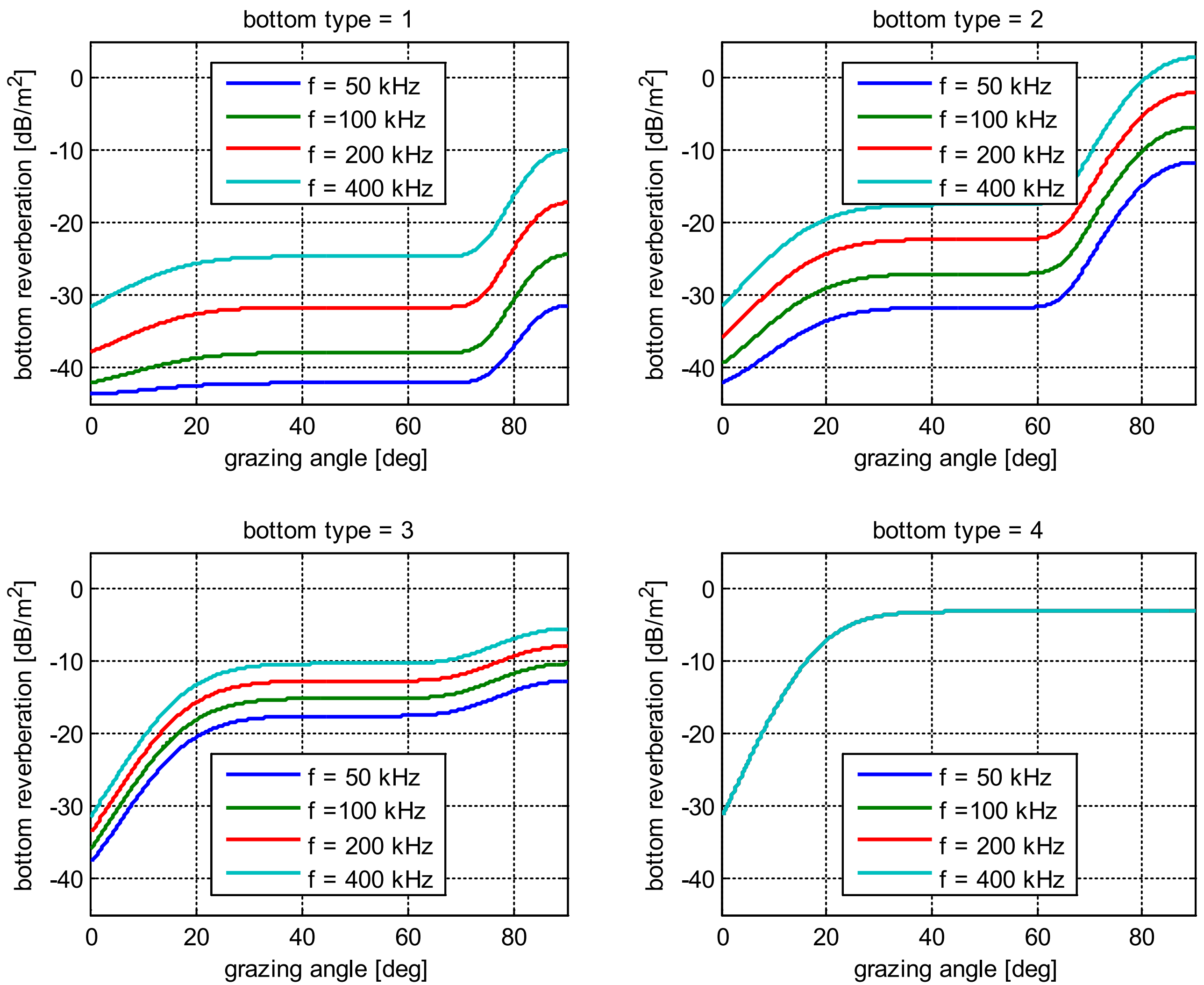
$bt = 3$ gravel $\quad$ $bt = 4$ rock.

In principle bt can be any real number satisfying

$$1 \le bt \le 4.$$

This allows an improved bottom type specification.


frequency = 50 kHz
frequency = 100 kHz
frequency = 200 kHz
frequency = 400 kHz
bottom reverberation [dB/m^2]
grazing angle [deg]
bt = 1
bt = 2
bt = 3
bt = 4
0
-10
-20
-30
-40
20
40
60
80


bottom type = 1
bottom type = 2
bottom type = 3
bottom type = 4
f = 50 kHz
f =100 kHz
f = 200 kHz
f = 400 kHz
bottom reverberation [dB/m2]
grazing angle [deg]
0
-10
-20
-30
-40
0
20
40
60
80

Volume Backscattering

In Section 1.3.2 we considered the frequency dependence of the sound attenuation which is partly caused by scattering in the water volume.

This also produces a backscattered sound field. However most volume reverberation is thought to arise from biological organisms and turbidity. The volume reverberation can be modeled by the so-called volume reverberation coefficient.

$$S_V = Sp + 7 \cdot \log_{10}(f) \quad [\mathrm{dB/m^3}],$$

where f and Sp indicate the frequency in kHz and the particle contribution in [dB/m^3]. The particle contribution parameter Sp is defined as follows

$$\left.\begin{array}{ll} Sp = -50 \text{ dB} & \text{High} \\ Sp = -70 \text{ dB} & \text{Moderate} \\ Sp = -90 \text{ dB} & \text{Low} \end{array}\right\} \text{Particle density}$$

Assignment 3:

Develop a Matlab program for computing the surface, bottom and volume reverberation coefficient.

Plot the coefficients S_S and S_B versus the grazing angle for various sets of (f, v_w) and (f, bt), respectively.

Plot the volume reverberation versus frequency for high, moderate and low particle densities.

Explain the results.

1.6 Ambient Noise

The isotropic noise level consists of the following components.

- Turbulence noise (1 Hz to 10 Hz)

$$NL_{\text{turb}}(f) = 30 - 30 \cdot \log_{10}(f), \quad f \ \text{ in } \ [\text{kHz}].$$

- Far shipping (traffic) noise (10 Hz to 300 Hz)

$$NL_{\text{traffic}}(f) = 10 \cdot \log_{10}\left(\frac{3 \cdot 10^8}{1 + 10^4 \cdot f^4}\right), \quad f \ \text{ in } \ [\text{kHz}].$$

- Sea state noise (300 Hz to 100 kHz)

$$NL_{\text{ss}}(f, v_w) = 40 + 10 \cdot \log_{10}\left(\frac{v_w^2}{1 + f^{5/3}}\right), \quad f \ \text{ in } \ [\text{kHz}],$$

where v_w denotes the wind speed in [kn].

- Thermal noise (molecular agitation) (100 kHz to 1 MHz)

  $$NL_{\text{therm}}(f) = -15 + 20 \cdot \log_{10}(f), \quad f \quad \text{in} \quad [\text{kHz}].$$

- Rainfall noise (1 kHz to 5 kHz)

  $$NL_{\text{rain}}(f, r_r) \quad \text{in} \quad [\text{dB}],$$

  where f and r_r denote the frequency and rate of rainfall, respectively.

- Biological noise (fishes, shrimps etc.)

  $$NL_{\text{bio}}(f, s) \quad \text{in} \quad [\text{dB}],$$

  where f and s denote the frequency and season, respectively.

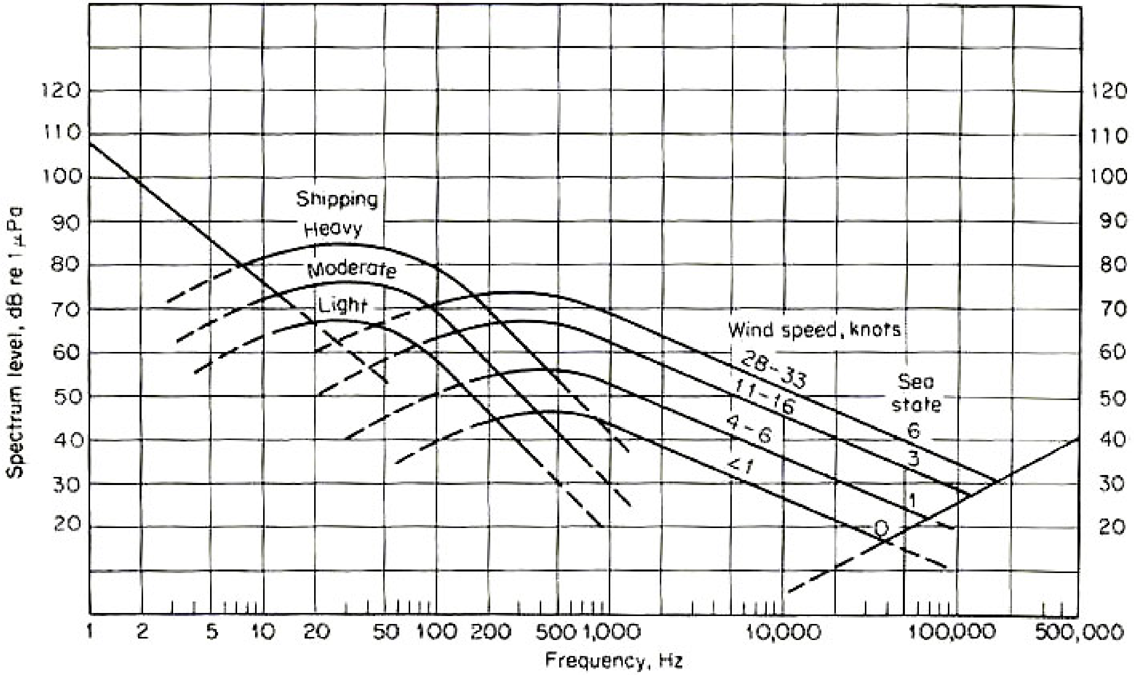
- Self (vessel) noise of sonar platform

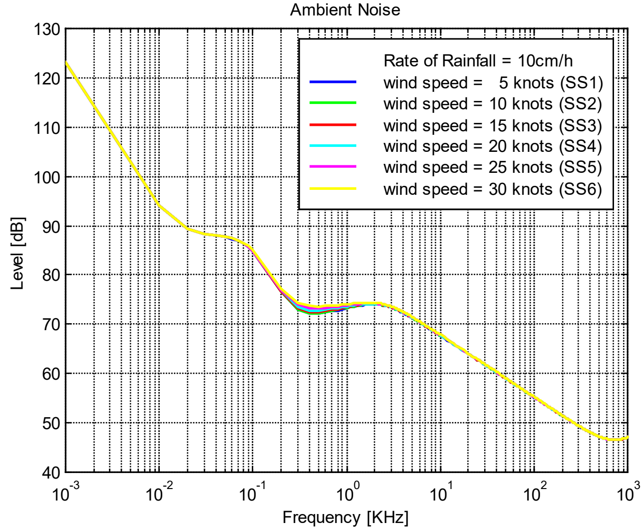
  $$NL_{\text{vessel}}(f, v_v) \quad \text{in} \quad [\text{dB}],$$

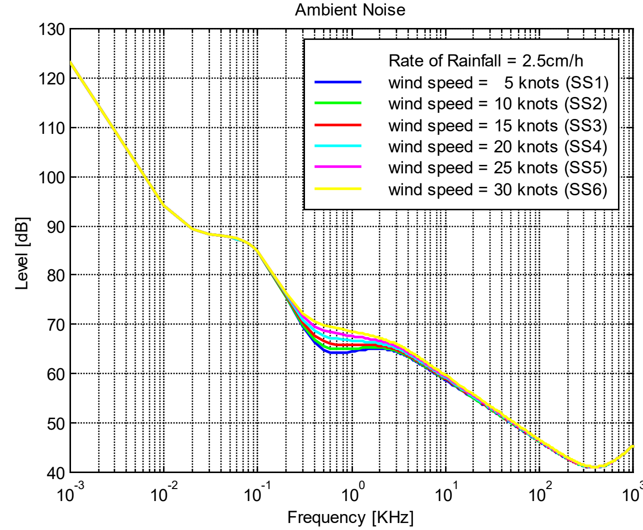
  where f and v_v denote the frequency and vessel speed, respectively.

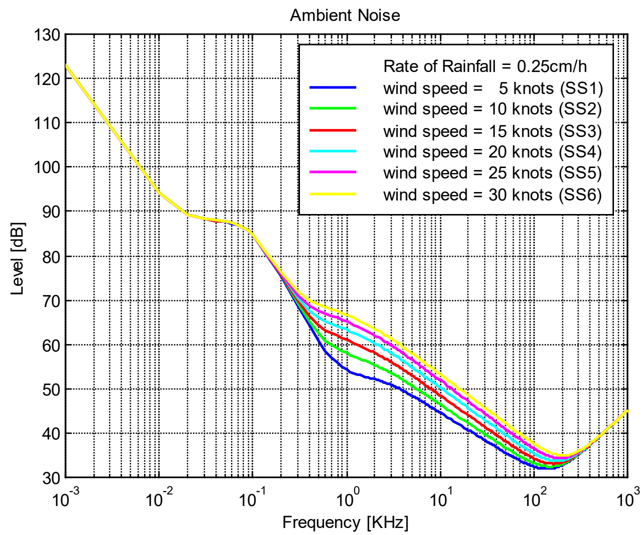
Thus the isotropic noise level can be determined by

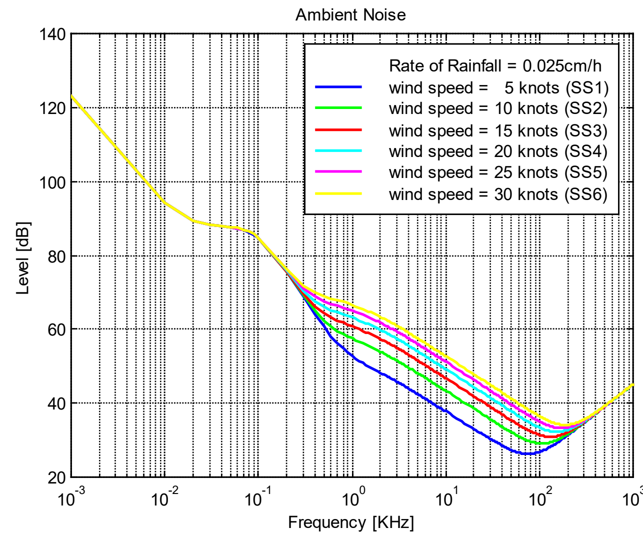
$$NL(f, v_w, r_r, s, v_v) = 10 \cdot \log_{10}(10^{0.1 \cdot NL_{\text{turb}}} + 10^{0.1 \cdot NL_{\text{traffic}}} + 10^{0.1 \cdot NL_{\text{ss}}} + 10^{0.1 \cdot NL_{\text{therm}}} + 10^{0.1 \cdot NL_{\text{rain}}} + 10^{0.1 \cdot NL_{\text{bio}}} + 10^{0.1 \cdot NL_{\text{vessel}}}).$$

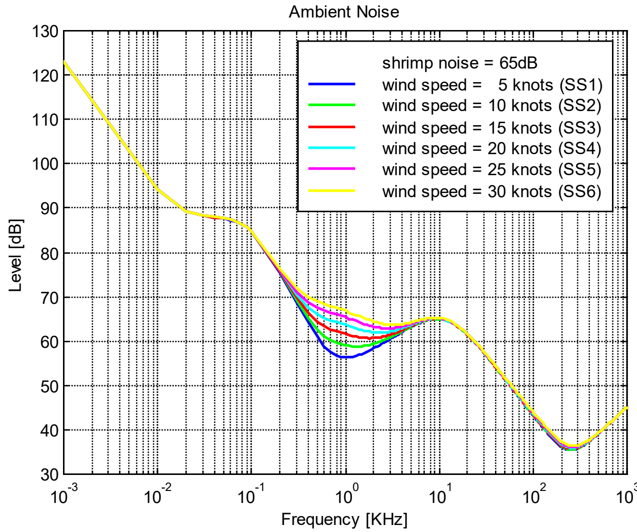


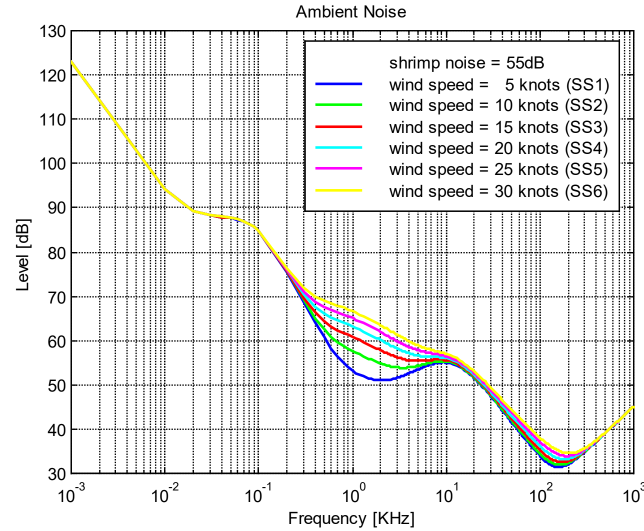

Ambient Noise
Rate of Rainfall = 10cm/h
wind speed = 5 knots (SS1)
wind speed = 10 knots (SS2)
wind speed = 15 knots (SS3)
wind speed = 20 knots (SS4)
wind speed = 25 knots (SS5)
wind speed = 30 knots (SS6)
Level [dB]
Frequency [KHz]

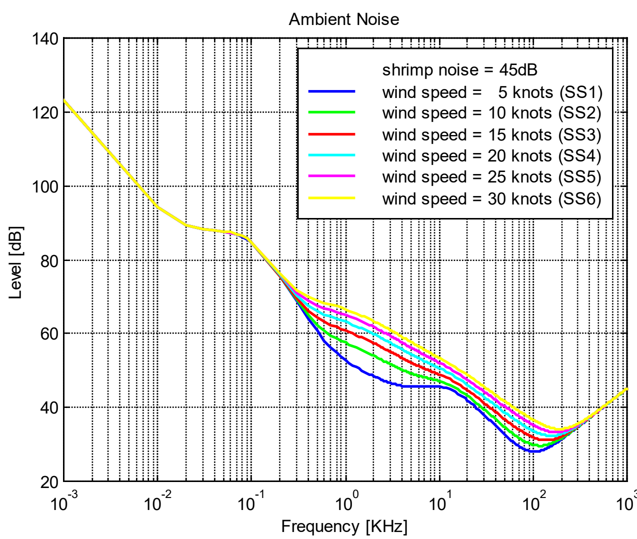

Ambient Noise
Rate of Rainfall = 2.5cm/h
wind speed = 5 knots (SS1)
wind speed = 10 knots (SS2)
wind speed = 15 knots (SS3)
wind speed = 20 knots (SS4)
wind speed = 25 knots (SS5)
wind speed = 30 knots (SS6)
Level [dB]
Frequency [KHz]

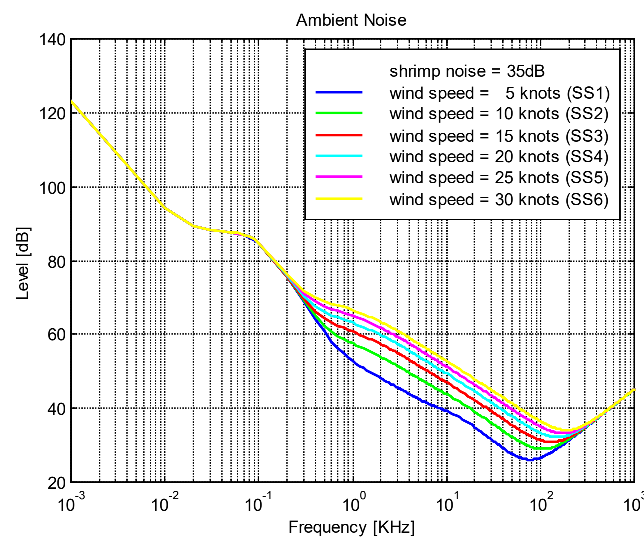

Ambient Noise
Rate of Rainfall = 0.25cm/h
wind speed = 5 knots (SS1)
wind speed = 10 knots (SS2)
wind speed = 15 knots (SS3)
wind speed = 20 knots (SS4)
wind speed = 25 knots (SS5)
wind speed = 30 knots (SS6)
Level [dB]
Frequency [KHz]


Ambient Noise
Rate of Rainfall = 0.025cm/h
wind speed = 5 knots (SS1)
wind speed = 10 knots (SS2)
wind speed = 15 knots (SS3)
wind speed = 20 knots (SS4)
wind speed = 25 knots (SS5)
wind speed = 30 knots (SS6)
Level [dB]
Frequency [KHz]


Ambient Noise
shrimp noise = 65dB
wind speed = 5 knots (SS1)
wind speed = 10 knots (SS2)
wind speed = 15 knots (SS3)
wind speed = 20 knots (SS4)
wind speed = 25 knots (SS5)
wind speed = 30 knots (SS6)
Level [dB]
Frequency [KHz]


Ambient Noise
shrimp noise = 55dB
wind speed = 5 knots (SS1)
wind speed = 10 knots (SS2)
wind speed = 15 knots (SS3)
wind speed = 20 knots (SS4)
wind speed = 25 knots (SS5)
wind speed = 30 knots (SS6)
Level [dB]
Frequency [KHz]


Ambient Noise
shrimp noise = 45dB
wind speed = 5 knots (SS1)
wind speed = 10 knots (SS2)
wind speed = 15 knots (SS3)
wind speed = 20 knots (SS4)
wind speed = 25 knots (SS5)
wind speed = 30 knots (SS6)
Level [dB]
Frequency [KHz]


Ambient Noise
shrimp noise = 35dB
wind speed = 5 knots (SS1)
wind speed = 10 knots (SS2)
wind speed = 15 knots (SS3)
wind speed = 20 knots (SS4)
wind speed = 25 knots (SS5)
wind speed = 30 knots (SS6)
Level [dB]
Frequency [KHz]

Assignment 4:

Develop a Matlab program for calculating the isotropic ambient noise level.

Plot the ambient noise level versus frequency for wind speeds of 5:5:30 kn and where NL_{vessel}, NL_{rain} and NL_{bio} are set to -999 dB.

Indicate the frequency domains where either NL_{traffic}, NL_{turb}, NL_{ss} or NL_{thermal} dominate.

1.7 Sonar Performance Prediction

1.7.1 Performance parameters

To assess the capabilities of a sonar system, parameters that measure the performance have to be defined, e.g.

EL: Echo Level
EE: Echo Excess
SN: Signal to Noise Ratio
SE: Signal Excess

1.7.2 Sound Propagation Related Parameters

Transmission Loss (*TL*)

The Transmission loss is given by

$$TL = \text{spreading loss} + \text{attenuation}, \quad [\text{dB}]$$

where TL is defined to be 0 dB on a sphere around the source of radius $R = 1$ m.

For a constant sound velocity profile and therefore spherical spreading the transmission loss can be determined by

$$TL(r,z) = 20 \cdot \log_{10}(R) + \alpha \cdot (R - 1\,\mathrm{m})$$

with

$$R = \sqrt{(r - r_0)^2 + (z - z_0)^2},$$

where r_0, z_0 denote the horizontal and vertical coordinates of the source locations and the receiver position, respectively.

In case of depth and range dependent cylinder symmetric sound velocity profiles the TL can be calculated by

$$TL(r,z) = 20 \cdot \log_{10}(R) + 10 \cdot \log_{10}\left(F(r,z)\right) + \alpha \cdot (R - 1\,\mathrm{m}),$$

where $F(r,z)$ denotes the so called focusing factor given by

$$F(r,z) = \frac{\text{actual spreading at } r,z}{\text{spherical spreading at } r,z}.$$

Isotropic Noise Level (*NL*)

The isotropic noise level $NL(f, v_w, r_r, s, v_v)$ describes for particular v_w, r_r, s and v_v the noise power within a 1 Hz band around frequency f.

Thus, assuming NL approximately white over the frequency band b of interest, the noise level is given by

$$NL_b = NL(f, v_w, r_r, s, v_v) + 10 \cdot \log_{10}(b).$$

Bottom Reverberation Strength (RS_B)

The bottom reverberation coefficient $S_B(f, bt, \vartheta)$ describes the reverberation strength of an insonified area of 1 m^2.

With c = sound speed,
τ = pulse length,
$2\theta_h$ = $\min(2\theta_{h,T}, 2\theta_{h,R})$,
$2\theta_{h,T}$ = horizontal 3 dB beam width of transmitter,
$2\theta_{h,R}$ = horizontal 3 dB beam width of receiver,
r_0, z_0 = coordinates of transmitter / receiver configuration,
r, z = coordinates of a particular point on the sea floor

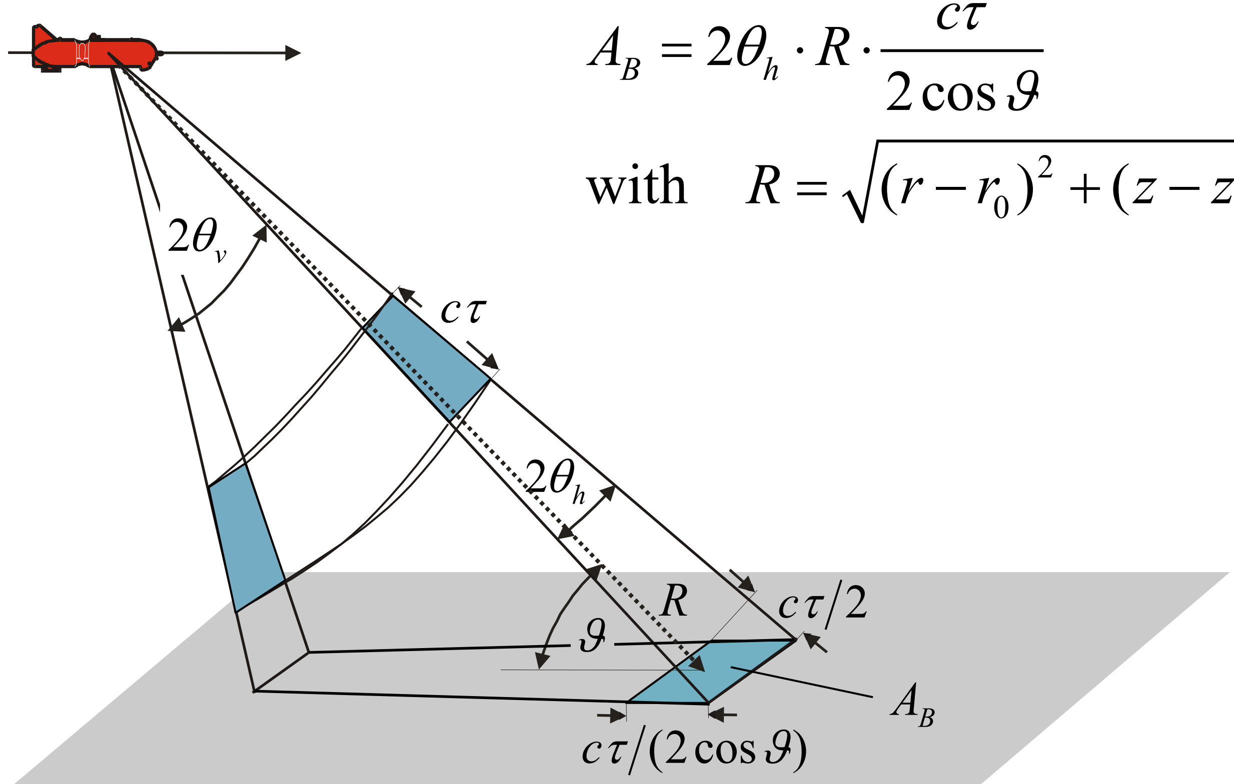
the bottom reverberation strength can be determined for given f and bt as function of ϑ by

$$RS_B = S_B(f, bt, \vartheta) + 10 \cdot \log_{10}(A_B),$$

where A_B denotes the insonified bottom area

$$A_B = 2\theta_h \cdot R \cdot \frac{c\tau}{2\cos\vartheta}$$

with $\quad R = \sqrt{(r - r_0)^2 + (z - z_0)^2}.$



Surface Reverberation Strength (RS_S)

Analog to the bottom reverberation strength, the surface reverberation strength is provided by

$$RS_S = S_S(f, v_w, \vartheta) + 10 \cdot \log_{10}(A_S),$$

where A_S denotes the insonified sea surface area

$$A_S = 2\theta_h \cdot R \cdot \frac{c\tau}{2\cos\vartheta} \quad \text{with} \quad R = \sqrt{(r - r_0)^2 + (z - z_0)^2}$$

and

and	c	=	sound speed,
	τ	=	pulse length,
	$2\theta_h$	=	min($2\theta_{h,T}$, $2\theta_{h,R}$),
	$2\theta_{h,T}$	=	horizontal 3 dB beam width of transmitter,
	$2\theta_{h,R}$	=	horizontal 3 dB beam width of receiver,
	r_0, z_0	=	coordinates of transmitter / receiver configuration,
	r, z	=	coordinates of a particular point on the sea surface.

Volume Reverberation Strength (RS_V)

The volume reverberation coefficient $S_V(Sp, f)$ describes the reverberation strength of an insonified volume of 1 m^3. Thus, the volume reverberation strength can be calculated by

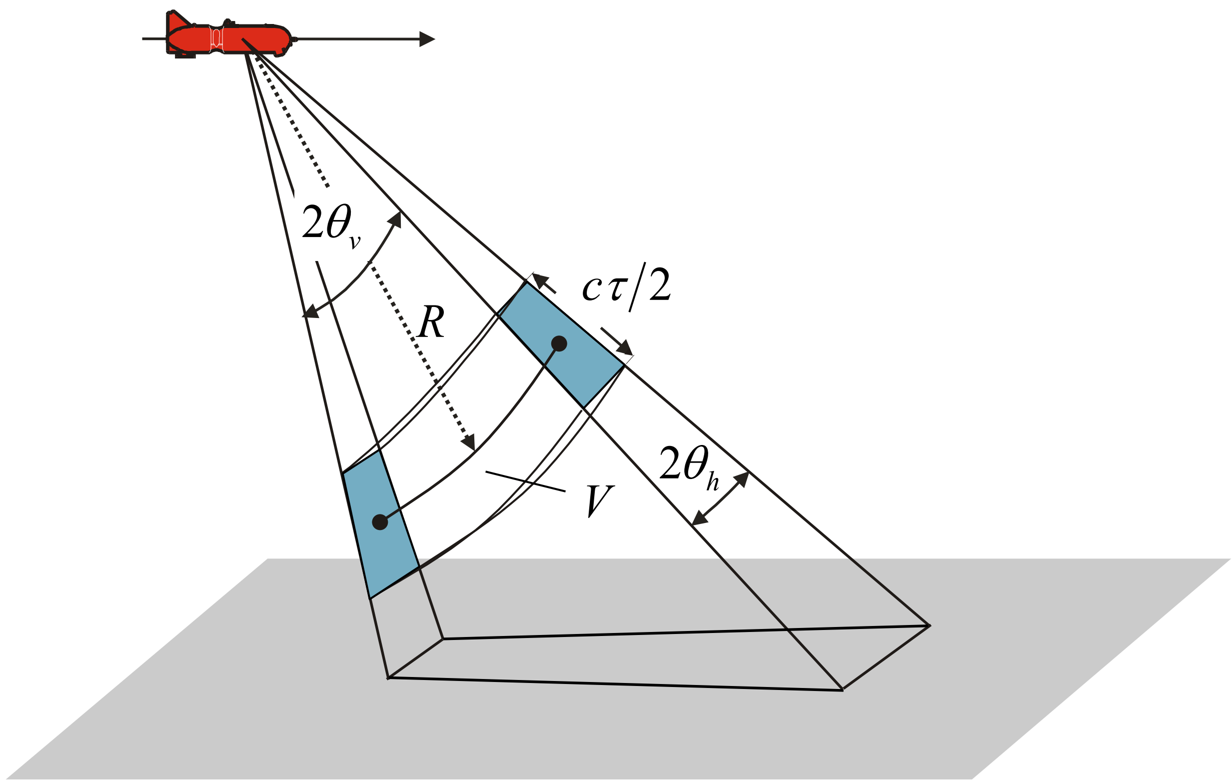
$$RS_V = S_V(Sp, f) + 10 \cdot \log_{10}(V),$$

where V denotes the insonified volume (isovelocity)

$$V = \frac{c\tau}{2} \cdot 2\theta_h \cdot 2\theta_v \cdot R^2 \quad \text{with} \quad R = \sqrt{(r - r_0)^2 + (z - z_0)^2}$$

and
c = sound speed,
τ = pulse length,
$2\theta_h$ = $\min(2\theta_{h,T}, 2\theta_{h,R})$,
$2\theta_{h,T}$ = horizontal 3 dB beam width of transmitter,
$2\theta_{h,R}$ = horizontal 3 dB beam width of receiver,
$2\theta_v$ = $\min(2\theta_{v,T}, 2\theta_{v,R})$,

$2\theta_{v,T}$ = vertical 3 dB beam width of transmitter,
$2\theta_{v,R}$ = vertical 3 dB beam width of receiver,
r_0, z_0 = coordinates of transmitter / receiver configuration,
r, z = coordinates of a particular point on the sea surface.



1.7.3 Sonar Equation

To determine the aforementioned reverberation levels the following environmental parameters have to be specified.

- bt: Bottom type
- v_w: Wind speed
- S: Salinity
- T : Water temperature
- c: Sound-speed-profile

For given sonar parameters, i.e.

- SL: Source Level in dB at 1m
- f : Center frequency of the sound signal
- b: Bandwidth of the sound signal
- τ: Pulse length of the sound signal

DI: Directivity Index of Receiver Array

BP_T: Transmitter Beam pattern (vertical)

BP_R: Receiver Beam pattern (vertical)

$2\theta_h$: Horizontal 3 dB Beamwidth $\min\{2\theta_{h,T}, 2\theta_{h,R}\}$

$2\theta_v$: Vertical 3 dB Beamwidth $\min\{2\theta_{v,T}, 2\theta_{v,R}\}$

and Target parameters, i.e.

TS: Target Strength

L_l: Target extent in lateral direction

L_r: Target extent in radial direction

the performance parameters can be determined by

$$
\begin{aligned}
EL(r,z) &= 10\cdot\log_{10}\left(el(r,z)\right) = 10\cdot\log_{10}\left(sl\cdot bp_{T,E}\cdot bp_{R,E}\cdot ts\big/ tl_E^2\right) \\
&= 10\cdot\log_{10}\left(10^{0.1\cdot SL}\cdot 10^{0.1\cdot BP_{T,E}}\cdot 10^{0.1\cdot BP_{R,E}}\cdot 10^{-0.2\cdot TL_E}\cdot 10^{0.1\cdot TS}\right) \\
&= SL + BP_{T,E} + BP_{R,E} - 2TL_E + TS,
\end{aligned}
$$

$$\begin{aligned} EE(r,z) &= 10\cdot\log_{10}\left(ee(r,z)\right) = 10\cdot\log_{10}\left(el\cdot di/nl_B\right) \\ &= 10\cdot\log_{10}\left(10^{0.1\cdot EL}\cdot 10^{-0.1\cdot(NL_B-DI)}\right) = EL-(NL_b-DI) \\ &= SL+BP_{T,E}+BP_{R,E}-2TL_E+TS-(NL_b-DI), \end{aligned}$$

$$\begin{aligned} SN(r,z) &= 10\cdot\log_{10}\left(sn(r,z)\right) = 10\cdot\log_{10}\left(el/til\right) \\ &= 10\cdot\log_{10}\left(10^{0.1\cdot EL}\cdot 10^{-0.1\cdot TIL)}\right) = EL-TIL \\ &= SL+BP_{T,E}+BP_{R,E}-2TL_E+TS-TIL \end{aligned}$$

and

$$\begin{aligned} SE(r,z) &= 10\cdot\log_{10}\left(se(r,z)\right) = 10\cdot\log_{10}\left(sn/dt\right) \\ &= 10\cdot\log_{10}\left(10^{0.1\cdot SN}\cdot 10^{-0.1\cdot DT)}\right) = SN-DT \\ &= SL+BP_{T,E}+BP_{R,E}-2TL_E+TS-TIL-DT, \end{aligned}$$

where DT denotes the detection threshold, TIL the total inference level

$$TIL(r,z) = 10 \cdot \log_{10}\left(til(r,z)\right) = 10 \cdot \log_{10}\left(nl_b / di + rl_B + rl_S + rl_V\right)$$

$$= 10 \cdot \log_{10}\left(10^{0.1 \cdot (NL_b - DI)} + 10^{0.1 \cdot RL_B} + 10^{0.1 \cdot RL_S} + 10^{0.1 \cdot RL_V}\right)$$

and RL_B, RL_S, and RL_V the reverberation level of the bottom, surface and volume, respectively, i.e.

$$RL_B = SL + BP_{T,B} + BP_{R,B} - 2TL_B + RS_B$$

$$RL_S = SL + BP_{T,S} + BP_{R,S} - 2TL_S + RS_S$$

$$RL_V = SL + \overline{BP_{T,V} + BP_{R,V} - 2TL_V} + RS_V \,.$$

For $c = \text{const.}$, we can write

$$TL_E = TL_B = TL_S = TL_V \,.$$

Furthermore, the following abbreviations have been used.

$$\left.\begin{array}{l} BP_{T,E} : \text{Transmitter} \\ BP_{R,E} : \text{Receiver} \end{array}\right\} \text{Beampattern value for the ray directed toward the target position}$$

$$\left.\begin{array}{l} BP_{T,B} : \text{Transmitter} \\ BP_{R,B} : \text{Receiver} \end{array}\right\} \text{Beampattern value for the ray directed toward the insonified bottom area}$$

$$\left.\begin{array}{l} BP_{T,S} : \text{Transmitter} \\ BP_{R,S} : \text{Receiver} \end{array}\right\} \text{Beampattern value for the ray directed toward the insonified surface area}$$

$$\left.\begin{array}{l} TL_E \\ TL_B \\ TL_S \\ TL_V \end{array}\right\} \text{Tranmission loss for the echo, bottom, surface and volume reverberation, respectively}$$

Assignment 5:

Develop a Matlab program for determining the $SN(r,z)$ and carry out calculations for the following parameters.

$z\,/\,r$:	up to 50 m / 600 m	bt:	mud, sand, gravel	v_w:	5, 15, 25 knots
S:	33 ppt	T:	15 °	c:	1480 m/s
SL:	220 dB re1μPa@1m	f:	100 kHz	τ:	100 μs
b:	10 kHz	BP_T:	0 dB (±90°)	BP_R:	0 dB (±90°)
DI:	30 dB	$2\theta_{h,R}$:	0.5°	$2\theta_{h,T}$:	90°
$2\theta_{v,R}$:	180°	$2\theta_{v,T}$:	180°	r_s:	0 m
z_s:	5 m				
TS:	−15 dB				

Discuss the observations, i.e. the impact of the bottom type and wind speed on the signal to noise ratio.

Literature

[1] Brekhovskikh, L.M.; Lysanov, Y.P.: Fundamentals of Ocean Acoustics, Springer, 2003

[2] Etter, P.C.: Underwater Acoustic Modeling, Spon Press, 2003

[3] Jensen, F.B: Computational Ocean Acoustics, Springer, 2011

[4] Lurton, X.: An Introduction to Underwater Acoustics, Springer, 2004

[5] Medwin, H.; Clay C.S: Acoustical Oceanography, Academic Press, 1998

[6] Tolstoy, I.; Clay C.S: Ocean Acoustics, AIP-Press, 1987

[7] Urick, R.I: Principles of Underwater sound, McGraw Hill, 1983